



# CBCS SCHEME

21MAT31

## Third Semester B.E./B.Tech. Degree Examination, June/July 2026 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find the Laplace transform of :

i)  $t e^{-t} \sin t$

ii)  $\frac{e^{-at} - e^{-bt}}{t}$ .

(06 Marks)

- b. Given  $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$  and  $f(t+a) = f(t)$  for all  $t > 0$ . Show that

$$f(s) = \frac{E}{s} \tanh(as/4).$$

(07 Marks)

- c. Using convolution theorem, find the inverse Laplace transform of  $\frac{s}{(s^2 + a^2)^2}$ . (07 Marks)

OR

- 2 a. Find the inverse Laplace transform of

$$\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}.$$

(06 Marks)

- b. Expression :

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$$

Interms of unit step functions and hence find its LT.

(07 Marks)

- c. Solve by using Laplace transform method :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, \text{ with } x(0) = 2 \text{ and } x'(0) = -1.$$

(07 Marks)

**Module-2**

- 3 a. Obtain the Fourier series of  $f(x) = \frac{\pi - x}{2}$  in  $0 < x < 2\pi$ . (06 Marks)

- b. Find half range Fourier cosine series of  $f(x) = (x - 1)^2$  in  $(0, 1)$  and hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad (07 \text{ Marks})$$

- c. Obtain the constant term and first sine and cosine terms in the Fourier expansion of  $y$  from the following table :

|   |   |    |    |    |    |    |
|---|---|----|----|----|----|----|
| x | 0 | 1  | 2  | 3  | 4  | 5  |
| y | 9 | 18 | 24 | 28 | 26 | 20 |

(07 Marks)

**OR**

- 4 a. Obtain the Fourier series of  $f(x) = |x|$ , in  $-\pi \leq x \leq \pi$ . (06 Marks)

- b. Obtain half-range Fourier sine series of

$$f(x) = \begin{cases} Kx, & 0 \leq x \leq \frac{l}{2} \\ K(l - x), & \frac{l}{2} \leq x \leq l \end{cases} \quad (07 \text{ Marks})$$

- c. Compute the first harmonics of the Fourier series of  $f(x)$  given the following table :

|   |     |         |          |       |          |          |        |
|---|-----|---------|----------|-------|----------|----------|--------|
| x | 0   | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
| y | 1.0 | 1.4     | 1.9      | 1.7   | 1.5      | 1.2      | 1.0    |

(07 Marks)

**Module-3**

- 5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx. \quad (06 \text{ Marks})$$

- b. Find the Fourier cosine transform of

$$f(t) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(07 Marks)

- c. Solve the difference equation  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  with  $y_0 = y_1 = 0$ , using z-transforms.

(07 Marks)

OR

- 6 a. Find the Fourier transform of  $f(x) = e^{-|x|}$ . (06 Marks)
- b. Find the z-transform of : i)  $(n+1)^2$  ii)  $\sin(3n+5)$ . (07 Marks)
- c. Find the inverse z-transform of  $\frac{20z^2+3z}{(5z-1)(5z+2)}$ . (07 Marks)

**Module-4**

- 7 a. Classify the following partial differential equations :
- i)  $x^2 u_{xx} + (1-y^2) u_{yy} = 0$ ,  $-\infty < x < \infty$ ,  $-1 < y < 1$
- ii)  $(1+x^2) u_{xx} + (5+2x^2) u_{xt} + (4+x^2) u_{tt} = 0$
- iii)  $(x+1) u_{xx} - 2(x+2) u_{xy} + (x+3) u_{yy} = 0$ . (10 Marks)
- b. Solve the equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  subject to the conditions  $u(x, 0) = \sin \pi x$ ,  $u(0, t) = u(1, t) = 0$ , carry out computations for two levels, taking  $h = \frac{1}{3}$ ,  $K = \frac{1}{36}$ . Using Schmidt's formula. (10 Marks)

OR

- 8 a. The transverse displacement  $u$  of a point at a distance  $x$  from one end and at any time ' $t$ ' of a vibrating string satisfies the equation  $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$  with boundary conditions  $u = 0$  at  $x = 0$ ,  $t > 0$  and  $u = 0$  at  $x = 4$ ,  $t > 0$  and initial conditions  $u = x(4-x)$  and  $\frac{\partial u}{\partial t} = 0$  at  $t = 0$ ,  $0 \leq x \leq 4$  solve this equation numerically for one half period of vibration, taking  $u = 1$  and  $K = \frac{1}{2}$ . (10 Marks)
- b. Find the values of  $u(x, t)$  satisfying the parabolic equation  $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$  and the boundary conditions  $u(0, t) = 0 = u(8, t)$  and  $u(x, 0) = 4x - \frac{1}{2}x^2$  at the points :  
 $x = i : i = 0, 1, 2, \dots$  and  $t = \frac{1}{8}j, j = 0, 1, 2, \dots$ . (10 Marks)

Module-5

- 9 a. Derive the Euler's equation. (06 Marks)
- b. Using R-K method, solve  $y'' = x(y')^2 - y^2$  for  $x = 0.2$ , correct to 4 decimal places given  $y(0) = 1, y'(0) = 0$ . (07 Marks)
- c. Find the extremals of the functional  $\int_0^{\pi/2} (y^2 - (y')^2 - 2y \sin x) dx$ , with  $y(0) = y(\pi/2) = 0$ . (07 Marks)

OR

- 10 a. Applying Milne's method, to solve :  $y'' + xy' + y = 0$  at  $x = 0.4$  given that  $y(0) = 1, y(0.1) = 0.9950, y(0.2) = 0.9802, y(0.3) = 0.9560$  and  $y'(0) = 0, y'$ . (07 Marks)
- b. Solve the variational problem  $\delta \int_0^{\pi/2} (y^2 - (y')^2) dx$  with  $y(0) = 0, y(\pi/2) = 2$ . (06 Marks)
- c. Show that geodesics on a plane is straight – line. (07 Marks)

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