



CBGS SCHEME

21MAT41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2026 Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Derive Cauchy–Riemann equation in Cartesian form. (06 Marks)
- b. Show that $w = z^n$ is analytic and hence find $\frac{dw}{dz}$. (07 Marks)
- c. Evaluate $\int_C z^2 dz$ along the curve made up of two line segments (i) from $z = 0$ to $z = 3$ and (ii) from $z = 3$ to $z = 3 + i$. (07 Marks)

OR

- 2 a. Find an analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$ by the Milne's – Thomson method. (06 Marks)
- b. State and prove Cauchy's Integral Theorem. (07 Marks)
- c. Evaluate $\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where 'c' is the circle $|z| = 3$. (07 Marks)

Module-2

- 3 a. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (06 Marks)
- b. Obtain the series solution of Bessel's differential equation :
$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0.$$
 (07 Marks)
- c. Express $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (07 Marks)

OR

- 4 a. Show that $J_{-\frac{1}{2}}(x) = J_{\frac{1}{2}}(x) \cot x$. (06 Marks)
- b. Find the series solution of the Legendre's equation
$$(1-x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0$$
 (07 Marks)
- c. Show that $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$. (07 Marks)

Module-3

- 5 a. Find the coefficient of correlation and line of regression of y on x for the data :

x :	1	2	3	4	5
y :	2	5	3	8	7

(06 Marks)

- b. If P is a pull required to lift a load W by means of a pulley block, find a linear law of the form $P = mW + C$ connecting P and W, using the following data:

P	12	15	21	25
W	50	70	100	120

where P and W are taken in kg. Also compute P when $W = 150$ kg.

(07 Marks)

- c. Using the method of least square, fit a curve $y = ax^b$ for the following data:

x :	3	4	5	6	7
y :	6	9	10	11	12

(07 Marks)

OR

- 6 a. The scores for 9 students in Physics and Mathematics are as follows:

Physics	35	23	47	17	10	43	9	6	28
Mathematics	30	33	45	23	8	49	12	4	31

Compute the ranks of students in the two subjects and compute the Spearman's rank correlation.

(06 Marks)

- b. If $2x - 3y = 0$ and $3x - 2y = 5$ are the lines of regression of the variables x and y. Calculate (i) Mean of x and y (ii) Coefficient of correlation between x and y and (iii) Standard deviation of x when variance of y is 2.

(07 Marks)

- c. Fit a second degree curve $y = ax^2 + bx + c$ for the data :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

Module-4

- 7 a. A random variable X has the following probability distribution :

X :	-2	-1	0	1	2	3
P(X) :	0.1	K	0.2	2K	0.3	K

- (i) Find K (ii) Evaluate $P(X < 1)$, $P(X > -1)$ (iii) Find its mean and standard deviation.

(06 Marks)

- b. Find the mean and S.D of Binomial Distribution.

(07 Marks)

- c. Alpha particles are emitted by a radioactive source at an average rate of 5 in a 20 minutes interval, using Poisson distribution, find the probability that there will be (i) exactly two emissions (ii) at least two emissions (iii) atmost two emissions, in a randomly chosen 20 minutes interval.

(07 Marks)

OR

- 8 a. A random variable X has density function

$$f(x) = \begin{cases} Kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

find K , Also, find $P(x \leq 2)$ and $P(x > 1)$.

(06 Marks)

- b. The probability that a news reader commits no mistake in reading the news is $1/e^3$. Using Poisson distribution find the probability that on a particular news broadcast he commits :
(i) Only 2 mistakes (ii) More than 3 mistakes (iii) Almost 3 mistakes. (07 Marks)
- c. The mean weight of 500 students at a certain school is 50 kgs and the S.D is 6 kgs. Assuming that the weights are normally distributed, find the expected number of students weighing between (i) 40 and 50 kgs (ii) More than 60 kgs. [$A(1.66) = 0.4522$] (07 Marks)

Module-5

- 9 a. The following probability density function

$$f(x) = \begin{cases} Kx^2, & -3 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find K and calculate $P(1 \leq x \leq 2)$, $P(x \leq 2)$ and $P(x > 1)$.

(06 Marks)

- b. A random sample of 10 boys had the following I.Q:
70, 120, 110, 101, 88, 83, 95, 98, 107, 100.
Does these data support the hypothesis that the population mean IQ's is 100 at 5% level of significance? [$t_{(0.05, 9)} = 2.26$]. (07 Marks)
- c. The following table gives the number of road accidents that occurred in a large city during the various days of a week. Test the hypothesis that the accidents are uniformly distributed over all the days of the week. [$\chi_{0.05}^2(6) = 12.59$]

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

(07 Marks)

OR

- 10 a. If joint probability distribution of X and Y is given by $f(x, y) = c(x^2 + y^2)$, for $x = -1, 0, 1, 3$; $y = -1, 2, 3$. Find (i) the value of c , (ii) $P(X = 0, Y \leq 2)$ (iii) $P(X \leq 1, Y > 2)$ (iv) $P(X + Y \geq 2)$. (06 Marks)

- b. The nine items of the sample have the following values :
45, 47, 50, 52, 48, 47, 49, 53 and 51

Does the mean of these differ significantly from the assumed mean of 47.5? [$t_{(0.05, 8)} = 2.31$]

(07 Marks)

- c. A set of 5 similar coins is tossed 320 times and the result is :

Number of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.

(07 Marks)
