

CBCS SCHEME

21MATCS41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2026 Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define Tautology. Prove that for any propositions p and q the compound proposition $[(\neg q) \wedge (p \rightarrow q)] \rightarrow (\neg p)$ is a tautology. (06 Marks)
- b. Prove the following logical equivalence :
 - i) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$
 - ii) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ (07 Marks)
- c. Give a direct proof and indirect proof for the statement "If n is an odd integer then $n + 9$ is an even integer". (07 Marks)

OR

- 2 a. Test the validity of arguments using rules of inference
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow s \\ r \rightarrow \neg s \\ \hline \neg p \vee r \\ \hline \therefore \neg p \end{array}$$
(06 Marks)
- b. If $p(x) = x^2 - 7x + 10 = 0$, $q(x) = x^2 - 2x - 3 = 0$, $r(x) : x < 0$
Determine the truth or falsity of the following statements when the universe U contains only the integers 2 and 5. If a statement is false, provide a explanation.
 - i) $\forall x, p(x) \rightarrow \neg r(x)$
 - ii) $\forall x, q(x) \rightarrow r(x)$
 - iii) $\exists x, q(x) \rightarrow r(x)$
 - iv) $\exists x, p(x) \rightarrow r(x)$ (07 Marks)
- c. Give an indirect proof of the statement
"The product of two even integers is an even integer". (07 Marks)

Module-2

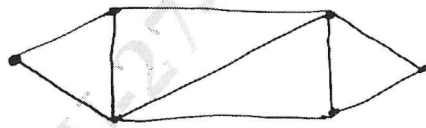
- 3 a. Let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = ax + b$, $g(x) = 1 - x + x^2$.
If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a and b . (06 Marks)
- b. Show that every simple graph of order ≥ 2 must have at least two vertices of the same degree. (07 Marks)

- c. Let $A = \{1, 2, 3, 4\}$ and let R be the relation on A defined by xRy if and only if "x divides y". Written x/y

- Write down R as a set of ordered pairs
- Draw the digraph of R
- Determine the in-degrees and out-degrees of the vertices in the digraph. (07 Marks)

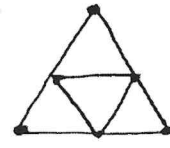
OR

- 4 a. Draw the Hasse diagram representing the positive divisors of 36. (06 Marks)
- b. Define: (i) Null graph (ii) Bipartite graph (iii) Euler circuit. Give an example for each. (07 Marks)
- c. Define isomorphic graphs. Determine whether the following graphs are isomorphic or not. [Refer Fig.Q4(c)(i) and Fig.Q4(c)(ii)]



(a)

Fig.Q4(c)(i)



(b)

Fig.Q4(c)(ii)

(07 Marks)

Module-3

- 5 a. Ten competitors in a beauty contest are ranked by three Judges A, B, C in the following order.

A	1	6	5	3	10	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	7	5

Use the rank correlation coefficient to discuss which pair of Judges have the nearest approach to common tastes of beauty. (06 Marks)

- b. Find the curve of the form $y = ax^b$ that best fit the following data:

x	3	4	5	6	7
y	6	9	10	11	12

(07 Marks)

- c. In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are eligible variance of $x = 9$, Regression equations : $8x - 10y + 66 = 0$, $40x - 18y = 214$. Find

- The Mean values of x and y .
- The Standard deviation of y .
- The Coefficient of correlation between x and y . (07 Marks)

OR

- 6 a. Calculate the coefficient of correlation and obtain the lines of regression of the following data:

x	1	2	3	4	5	6	7	8
y	9	8	10	12	11	13	14	16

Also compute y corresponds to $x = 6.2$

(06 Marks)

- b. Fit a curve $y = a + bx + cx^2$ to the following data:

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

(07 Marks)

- c. Determine rank correlation for the following data which shows the marks obtained in two quizzes in mathematics.

Marks in I Quiz (x)	6	5	8	8	7	6	10	4	9	7
Marks in II Quiz (y)	8	7	7	10	5	8	10	6	8	6

(07 Marks)

Module-4

- 7 a. The probability density function of a variate X is

x	0	1	2	3	4	5	6
p(x)	K	3k	5k	7k	9k	11k	13k

Find the value of k and evaluate (i) $p(x < 4)$ (ii) $p(x \geq 5)$ (iii) $p(3 \leq x \leq 6)$ (06 Marks)

- b. Derive the mean and variance of the binomial distribution. (07 Marks)

- c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75 ($A(1) = 0.3413$). (07 Marks)

OR

- 8 a. Find the constant k such that

$$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function. Compute :

- (i) $p(1 \leq x \leq 2)$ (ii) $p(x \leq 2)$ (iii) $p(x > 1)$ (06 Marks)

- b. The probability that an individual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals more than 2 will get a bad reaction. (07 Marks)

- c. In an examination 7% of students scores less than 35% marks and 89% of students score less than 60% marks. Find the mean and SD if the marks are normally distributed it is given that $A(1.2268) = 0.39$ and $A(1.4757) = 0.43$. (07 Marks)

Module-5

- 9 a. The joint probability distribution of two random variables X and Y are given below:

	Y	-3	2	4
X				
1		0.1	0.2	0.2
2		0.3	0.1	0.1

Determine : (i) $E(X)$ and $E(Y)$ (ii) $E(XY)$

(iii) $COV(X, Y)$ (iv) $P(X, Y)$

(06 Marks)

- b. Find the student 't' for the following variables values in a sample of eight -4, -2, -2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. (07 Marks)
- c. Genetic theory states that children having one parent of blood type M and the other of blood type N will always be one of the three M, MN, N and that the proportions of these types will be on an average 1 : 2 : 1. A report states that out of 300 childrens having one M 45% of type MN and the remaining of type N. Test the theory by χ^2 test. ($\chi^2_{0.05} = 5.99$) (07 Marks)

OR

- 10 a. Define the terms : (i) Null Hypothesis and Alternative Hypothesis
(ii) Type-I and Type -II Errors
(iii) Lead of Significance (06 Marks)

- b. The joint distribution of two random variables X and Y is given by the following table:

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of X and Y. Also verify that X and Y are independent. (07 Marks)

- c. The nine items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5 ($t_{0.05} = 2.31$) (07 Marks)
