



CBCS SCHEME

21MAT31

Third Semester B.E./B.Tech. Degree Examination, June/July 2026 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of :

i) $t e^{-t} \sin t$

ii) $\frac{e^{-at} - e^{-bt}}{t}$

(06 Marks)

b. Given $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ and $f(t+a) = f(t)$ for all $t > 0$. Show that

$$f(s) = \frac{E}{s} \tanh(as/4).$$

(07 Marks)

c. Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$. (07 Marks)

OR

2 a. Find the inverse Laplace transform of

$$\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}.$$

(06 Marks)

b. Expression :

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$$

Interms of unit step functions and hence find its LT.

(07 Marks)

c. Solve by using Laplace transform method :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, \text{ with } x(0) = 2 \text{ and } x'(0) = -1.$$

(07 Marks)

Module-2

- 3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. (06 Marks)

- b. Find half range Fourier cosine series of $f(x) = (x - 1)^2$ in $(0, 1)$ and hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

(07 Marks)

- c. Obtain the constant term and first sine and cosine terms in the Fourier expansion of y from the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

OR

- 4 a. Obtain the Fourier series of $f(x) = |x|$, in $-\pi \leq x \leq \pi$. (06 Marks)

- b. Obtain half-range Fourier sine series of

$$f(x) = \begin{cases} Kx, & 0 \leq x \leq \frac{l}{2} \\ K(l - x), & \frac{l}{2} \leq x \leq l \end{cases}$$

(07 Marks)

- c. Compute the first harmonics of the Fourier series of $f(x)$ given the following table :

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(07 Marks)

Module-3

- 5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx.$$

(06 Marks)

- b. Find the Fourier cosine transform of

$$f(t) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(07 Marks)

- c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$, using z-transforms.

(07 Marks)

OR

- 6 a. Find the Fourier transform of $f(x) = e^{-|x|}$. (06 Marks)
- b. Find the z-transform of : i) $(n+1)^2$ ii) $\sin(3n+5)$. (07 Marks)
- c. Find the inverse z-transform of $\frac{20z^2 + 3z}{(5z-1)(5z+2)}$. (07 Marks)

Module-4

- 7 a. Classify the following partial differential equations :
- i) $x^2 u_{xx} + (1-y^2) u_{yy} = 0$, $-\infty < x < \infty$, $-1 < y < 1$
- ii) $(1+x^2) u_{xx} + (5+2x^2) u_{xt} + (4+x^2) u_{tt} = 0$
- iii) $(x+1) u_{xx} - 2(x+2) u_{xy} + (x+3) u_{yy} = 0$. (10 Marks)
- b. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x, 0) = \sin \pi x$, $u(0, t) = u(1, t) = 0$, carry out computations for two levels, taking $h = \frac{1}{3}$, $K = \frac{1}{36}$. Using Schmidt's formula. (10 Marks)

OR

- 8 a. The transverse displacement u of a point at a distance x from one end and at any time ' t ' of a vibrating string satisfies the equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u = 0$ at $x = 0$, $t > 0$ and $u = 0$ at $x = 4$, $t > 0$ and initial conditions $u = x(4-x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$, $0 \leq x \leq 4$ solve this equation numerically for one half period of vibration, taking $u = 1$ and $K = \frac{1}{2}$. (10 Marks)
- b. Find the values of $u(x, t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0, t) = 0 = u(8, t)$ and $u(x, 0) = 4x - \frac{1}{2}x^2$ at the points : $x = i : i = 0, 1, 2, \dots$ and $t = \frac{1}{8}j, j = 0, 1, 2, \dots, 5$. (10 Marks)

Module-5

- 9 a. Derive the Euler's equation. (06 Marks)
- b. Using R-K method, solve $y'' = x(y')^2 - y^2$ for $x = 0.2$, correct to 4 decimal places given $y(0) = 1, y'(0) = 0$. (07 Marks)
- c. Find the extremals of the functional $\int_0^{\pi/2} (y^2 - (y')^2 - 2y \sin x) dx$, with $y(0) = y(\pi/2) = 0$. (07 Marks)

OR

- 10 a. Applying Milne's method, to solve : $y'' + xy' + y = 0$ at $x = 0.4$ given that $y(0) = 1, y(0.1) = 0.9950, y(0.2) = 0.9802, y(0.3) = 0.9560$ and $y'(0) = 0, y'$. (07 Marks)
- b. Solve the variational problem $\delta \int_0^{\pi/2} (y^2 - (y')^2) dx$ with $y(0) = 0, y(\pi/2) = 2$. (06 Marks)
- c. Show that geodesics on a plane is straight – line. (07 Marks)

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