USN | | | | | | | | | | | **13MCA12**

First Semester MCA Degree Examination, June/July 2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Prove that for any propositions p, q, r the compound proposition $[(p\to q)\wedge(q\to r)]\to(p\to r)$ is tautology. **(08 Marks)**

b. Examine whether the compound propositions,
$[(p\vee q)\to r]\longleftrightarrow[\sim r\to\sim(p\vee q)]$ are logically equivalent. **(08 Marks)**

c. Prove the De Morgan law $\sim(p\vee q)\Leftrightarrow\sim p\wedge\sim q$. **(04 Marks)**

2 a. Mention various rules of Inferences. **(08 Marks)**

b. Find whether the following is a valid argument for which the universe is set of all students
No Engineering Student is bad in studies.
Anil is not bad in studies
∴ Anil is an Engineering student **(08 Marks)**

c. Negate and simplify each of the following :
(i) $\exists x, [p(x)\vee q(x)]$ (ii) $\forall x, [p(x)\wedge\sim q(x)]$
(iii) $\forall x, [p(x)\to q(x)]$ (iv) $\forall x, [\{p(x)\vee q(x)\}\to r(x)]$ **(04 Marks)**

3 a. If A, B, C are finite sets, prove that
$|A\cup B\cup C| = |A|+|B|+|C|-|A\cap B|-|B\cap C|-|A\cap C|+|A\cap B\cap C|$ **(08 Marks)**

b. Find the sets A and B if
$A-B=\{1,3,7,11\}$, $B-A=\{2,6,8\}$ and $A\cap B=\{4,9\}$ **(04 Marks)**

c. Find the number of ways of giving 10 identical gift boxes to 6-persons A, B, C, D, E, F in such a way that the total number of boxes given to A and B together does not exceed 4. **(08 Marks)**

4 a. Prove by mathematical induction that for every positive integer n, 5 divides n^5-n. **(08 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1=7$, $a_n=2a_{n-1}+1$ for $n\geq 2$. **(06 Marks)**

c. If $27=189m+243n$, find m and n. Show that m and n are not unique. **(06 Marks)**

5 a. Define injective function, bijective function and surjective functions. Give one example for each. **(06 Marks)**

b. State and prove generalized pigeon hole principle show that if 11 numbers are chosen from the set {1, 2, 3,……..20}, one of them is a multiple of another. **(06 Marks)**

c. Let f and g be functions from R to R, defined by $f(x)=ax+b$, $g(x)=1-x+x^2$. If $(gof)(x)=9x^2-9x+3$, determine a, b. **(08 Marks)**

6 a. If A = {1, 2, 3, 4} R, S are relations on A, defined by
R = {(1, 2), (1, 3), (2, 4), (4, 4)}
S = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)}
Find (i) RoS (ii) SoR (iii) R^2 (iv) S^2. Write the matrices. **(10 Marks)**

b. Let A = {1, 2, 3, 4, 5} define a relations R on A×A, by $(x_1,y_1)R(x_2,y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.
(i) Verify R is an equivalence relation on A×A
(ii) Determine the equivalence classes [(1, 3)] [(2, 4)] and [(1, 1)]
(iii) Determine partition of A×A induced by R. **(10 Marks)**

7 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined on A defined by aRb if and only if a is multiple of b. Represent the relation R as a matrix and draw its digraph. **(08 Marks)**
b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**
c. Given G = G(V, E), G′ = G′(V′, E′) be two graphs and $f : G \to G'$ be an isomerism prove that $f^{-1} : G' \to G$ is also an isomerism. **(06 Marks)**

8 a. Using the merge-sort method, sort the list 7, 3, 8, 4, 5, 10, 6, 2, 9. **(08 Marks)**
b. Find the chromatic numbers of the following graph shown in Fig.Q8(b). **(06 Marks)**

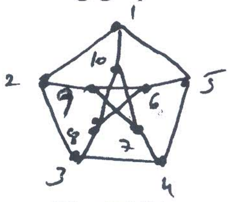

Fig.Q8(b)

c. Define : (i) Complete graph (ii) Bipartite graph (iii) Complete Bipartite graph. **(06 Marks)**

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