ACHARYA INSTITUTE OF TECHNOLOGY
Bangalore - 560090

USN | | | | | | | | | | |

10MAT11

First Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y = \frac{1}{2x+1}$ then the 10^{th} derivative of y is

A) $\frac{2^{10}10!}{(-2x+1)^{11}}$ B) $\frac{2^{10}10!}{(2x+1)^{11}}$ C) $\frac{2^{10}10!}{(2x-1)^{11}}$ D) $\frac{2^{10}10!}{(2x+1)^{-11}}$

ii) If $y = \sin 2x$ then y_n is

A) $2^n \sin\left(2x + \frac{n\pi}{2}\right)$ B) $2^n \cos\left(2x + \frac{n\pi}{2}\right)$ C) $2^n \sin\left(2x - \frac{n\pi}{2}\right)$ D) none of these

iii) If f(x) is continuous in [a, b], differentiable in (a, b) and f(a) = f(b), then there exist atleast one point $c \in (a, b)$ such that $f'(c)$ is equal to

A) 0 B) −1 C) $\frac{f(b)-f(a)}{b-a}$ D) $\frac{f(b)-f(a)}{a-b}$

iv) Maclaurin's series expansion of e^x is

A) $1 + 2x + \frac{x^2}{2} + ...$ B) $1 + x + \frac{x^2}{2} + ...$ C) $1 - 2x + \frac{x^2}{2} - ....$ D) $1 - x + \frac{x^2}{2} - ....$

b. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^p$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + p^2)y_n = 0$. **(06 Marks)**

c. State Rolle's theorem and verify the theorem for the function $f(x) = \log\left(\frac{x^2 + ab}{x(a+b)}\right)$ in [a, b], $b > a > 0$. **(05 Marks)**

d. Find the Maclaurin's series expansion of $\log(1 + e^x)$ upto the term containing x^4. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\lim_{x \to 0} \frac{a^x - b^x}{x}$ is

A) $\log\left(\frac{b}{a}\right)$ B) $\log\left(\frac{a}{b}\right)$ C) $\log(a - b)$ D) 1

ii) Angle between radius vector and tangent to the curve $r = a \sin\theta$ is

A) θ B) −θ C) $\frac{\pi}{2} - \theta$ D) $\frac{\pi}{2} + \theta$

iii) The radius of curvature of any point on the curve $x = a\cos\theta$ and $y = a\sin\theta$ is

A) $a \sin\theta$ B) θ C) $\frac{\theta}{2}$ D) a

iv) The derivative of arc length $\frac{ds}{d\theta}$ for the polar curve $r = f(\theta)$ is

A) $\sqrt{r^2 + \frac{d^2r}{d\theta^2}}$ B) $\sqrt{r + \left(\frac{dr}{d\theta}\right)^2}$ C) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{r^2 + \left(\frac{d\theta}{dr}\right)^2}$

b. Evaluate the following:

i) $\lim_{x \to 0} (1+x)^{1/x}$ ii) $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$ **(06 Marks)**

c. For the curve $y = \frac{ax}{a+x}$, if ρ is the radius of curvature at any point (x, y), show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$. **(05 Marks)**

d. Find the angle of intersection of the following pair of curves $r = a\log\theta$, $r = \frac{a}{\log\theta}$. **(05 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = ax^2 + by^2 + abxy$, then $\frac{\partial^3 u}{\partial x^2 \partial y}$ is

A) zero B) $a+b+ab$ C) ab D) none of these

ii) If $u = x^4 y^5$, where $x = t^2$ and $y = t^3$, then $\frac{du}{dt}$ is

A) $22\,t^{23}$ B) $20\,t^{19}$ C) $9\,t^8$ D) $23\,t^{22}$

iii) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

A) $r^2 \sin 2\theta$ B) r^2 C) r D) $r \sin 2\theta$

iv) The necessary condition for u = f(x, y) to be extremal is

A) $u_x \neq 0, u_y \neq 0$ B) $u_x = 0, u_y = 0$ C) $u_x > 0, u_y > 0$ D) $u_x < 0, u_y < 0$

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0) is 20. **(06 Marks)**

c. If $z = \cos(x+ay) + \sin(x-ay)$ prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$. **(05 Marks)**

d. The deflection at the centre of a rod of length ℓ and diameter d, supported at its ends and located at the centre a weight w, which varies as $w\ell^3 d^{-4}$. Determine the percentage increase in w, ℓ and d of 5, 4 and 3 respectively. **(05 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\vec{F} = 3x^2\hat{i} - xy\hat{j} + (a-3)xz\hat{k}$ is solenoidal, then a is

A) 0 B) -2 C) 2 D) 3

ii) If $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, then curl $\vec{A}$ is given by

A) $2x\hat{i} + 2y\hat{j} + 2z\hat{k}$ B) 0 C) $\frac{x\hat{i} + y\hat{j} + z\hat{k}}{2}$ D) $2x + 2y + 2z$

iii) If $\phi = xy + yz + zx$, then grad ϕ at (1, 1, 1) is

A) $2\hat{i} + 2\hat{j} + 2\hat{k}$ B) 0 C) $\hat{i} + \hat{j} + \hat{k}$ D) $3\hat{i} + 3\hat{j} + 3\hat{k}$

iv) The gradient of a scalar field is a

A) vector B) scalar C) constant D) none of these

b. If $\vec{F} = (x+y+z)\hat{i} + \hat{j} - (x+y)\hat{k}$ then show that $\vec{F} \cdot \text{curl}\,\vec{F} = 0$. **(06 Marks)**

c. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\vec{F}$ is irrotational. **(05 Marks)**

d. Derive an expression for div $\vec{F}$ in orthogonal curvilinear coordinates. **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The Leibnitz's rule for differentiation under the integral sign is

A) $\phi'(y) = \int_a^b \frac{\partial}{\partial y}[f(x, y)]dx$ B) $\phi'(y) = \int_a^b \frac{\partial}{\partial x \partial y}[f(x, y)]dx$

C) $\phi(y) = \int_a^b \frac{\partial}{\partial x}[f(x, y)]dx$ D) none of these

ii) The value of $\int_0^{\pi/2} \sin^6 x dx$ is

A) $\frac{5\pi}{8}$ B) $\frac{5\pi}{64}$ C) $\frac{5\pi}{32}$ D) $\frac{5\pi}{16}$

iii) The value of $\int_0^{\pi/2} \sin^5 x \cos^5 x \, dx$ is

A) $\frac{1}{90}$ B) $\frac{1}{60}$ C) $\frac{1}{30}$ D) $\frac{1}{70}$

iv) Surface area of a solid of revolution of the curve y = f(x), if rotated about x-axis is

A) $\int_{x=a}^{b} 2\pi y dx$ B) $\int_{x=a}^{b} 2\pi x dy$ C) $\int_{x=a}^{b} 2\pi y ds$ D) $\int_{x=a}^{b} 2\pi x ds$

b. Using the rule of differentiation under the integral sign, evaluate $\int_0^{\pi} \frac{\log(1 + \alpha \cos x)}{\cos x} dx$. **(06 Marks)**

c. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$. **(05 Marks)**

d. Find the area of the Cardioid $r = a(1 + \cos\theta)$. **(05 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The solution of $\frac{dy}{dx} + \frac{y}{x} = 0$ is

A) $\frac{y}{x} = c$ B) $\frac{x}{y} = c$ C) $x - y = c$ D) $xy = c$

ii) The orthogonal trajectory of the family of lines y = ax is

A) $x^2 + y^2 = c^2$ B) $x^2 - y^2 = c^2$ C) $xy = c$ D) $\frac{x}{y} = c$

iii) The solution of the differential equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ is

A) $y = \log x + c$ B) $y = x \log x + c$ C) $y = x(\log x + c)$ D) none of these

iv) The general solution of the differential equation $(x - y)dx - (x - y)dy = 0$ is

A) $\frac{x^2}{2} - y - \frac{y^2}{2} = c$ B) $\frac{x^2}{2} - y + \frac{y^2}{2} = c$ C) $\frac{x^2}{2} - yx + \frac{y^2}{2} = c$ D) none of these

b. Solve $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$. **(06 Marks)**

c. Solve $x \log x \frac{dy}{dx} + y = 2 \log x$. **(05 Marks)**

d. Find the orthogonal trajectories of the family $x^{2/3} + y^{2/3} = a^{2/3}$.

7 a. Choose the correct answers for the following : **(04 Marks)**

i) The system of equations $AX = B$ is consistent if

A) $\rho(A) = \rho([A:B])$ B) $\rho(A) = \rho(B)$

C) $\rho(A) = \rho([B:A])$ D) all of these

ii) The system of equations $AX = 0$ is always

A) inconsistent B) consistent C) both A and B D) none of these

iii) Which of the following is in the normal form

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ D) all of these

iv) The rank of the matrix $\begin{bmatrix} 41 & 42 & 43 \\ 42 & 43 & 44 \\ 43 & 44 & 45 \end{bmatrix}$ is

A) 0 B) 2 C) 1 D) 3

b. Reduce the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank. **(06 Marks)**

c. Find the value of λ such that the system $2x - y + \lambda z = 0$, $3x + 2y + (\lambda - 2)z = 0$, $x - 4y + 5z = 0$ has non-trivial solution and hence solve the system for λ. **(05 Marks)**

d. Solve $x + y + z = 1$, $4x + 3y - z = 6$, $3x + 5y + 3z = 4$ by Gauss Jordon method. **(05 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) The eigen values of the matrix A exists, if A is a

A) rectangular matrix B) any matrix

C) null matrix D) square matrix

ii) The eigen values of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ are

A) 1, 3 B) 1, 6 C) 1, 5 D) 1, 4

iii) Which of these is in quadratic form

A) $x^2 + y^2 + z^2 - 2xy + yz - zx$ B) $x^3 + y^3 + z^2$

C) $(x - y + z)^2$ D) both A and C

iv) The quadratic form $(X'AX)$ is positive definite if

A) All the eigen values of $A > 0$

B) Atleast one eigen value of A is > 0

C) All eigen values are > 0 and atleast one eigen value is 0

D) No such condition

b. Reduce the matrix $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ to diagonal form. Hence find A^6. **(06 Marks)**

c. Show that the linear transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular write down the inverse transformation. **(05 Marks)**

d. Reduce the quadratic form $3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$ to canonical form and indicate its nature, rank, index and signature. **(05 Marks)**

* * * * *