ACHARYA INSTITUTE OF TECHNOLOGY
Bangalore - 560090

USN | | | | | | | | | | | **14MAT11**

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. Find the n^{th} derivative of $y = \sin^2 x \sinh^2 x + \log_{10}(x^2 - 3x + 2)$. **(07 Marks)**

b. Find the pedal equation for the curve $r = a + b\cos\theta$. **(06 Marks)**

c. Obtain radius of curvature for the parametric curve, $x = a(t - \sin t)$ $y = a(1 - \cos t)$. **(07 Marks)**

2 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$. Hence obtain $y_n(0)$. **(07 Marks)**

b. Find the angle of intersection between the curves $r = 2\sin\theta$; $r = 2(\sin\theta + \cos\theta)$. **(06 Marks)**

c. Find the radius of curvature for the polar curve $r^2 = a^2\cos 2\theta$. **(07 Marks)**

Module-2

3 a. Evaluate : $\lim\limits_{x\to 0}(\cos x)^{\cot^2 x}$. **(06 Marks)**

b. Determine Maclarin's series for the function for $f(x) = \log(1 + \cos x)$ upto term containing x^4. **(07 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then obtain the value of $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$. **(07 Marks)**

4 a. Find total derivative of u with respect to t where $u = \tan^{-1}x/y$, $x = e^t - e^{-t}$, $y = e^t + e^{-t}$. **(06 Marks)**

b. If $u = \frac{x}{y - z}$, $v = \frac{y}{z - x}$, $w = \frac{z}{x - y}$, find the Jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. Determine whether u, v and w are functionally dependent. **(07 Marks)**

c. If x y z be the angles of a triangle, show that the maximum value of sin x sin y sin z is $\frac{3\sqrt{3}}{8}$. **(07 Marks)**

Module-3

5 a. A particle moves along $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$, where 't' denotes time. Find the magnitudes of velocity and acceleration at time $t = 2$. **(07 Marks)**

b. Assuming the validity of differentiation under integral sign prove that
$$\int_0^\infty e^{-x^2}\cos\alpha x\,dx = \frac{\sqrt{\pi}}{2}e^{-\alpha^2/4}.$$ **(07 Marks)**

c. Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$, using general rules of tracing the curve. **(06 Marks)**

6 a. If $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ find curl $\vec{F}$. Is $\vec{F}$ irrotational? **(07 Marks)**

b. Prove that if $\vec{F}$ is a vector point function div (curl $\vec{F}$) = 0. **(07 Marks)**

c. If $\vec{r}$ is a position vector of a point in space obtain div $\vec{r}$ and curl $\vec{r}$. **(06 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(07 Marks)**

b. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\, dx$, where 'n' is a positive integer. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(06 Marks)**

8 a. Show that family $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ with λ as a parameter is self orthogonal. **(07 Marks)**

b. Evaluate : $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx$. **(07 Marks)**

c. Solve : $(y^2e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} + 3y^2)\,dy = 0$. **(06 Marks)**

Module-5

9 a. Solve by gauss elimination method :
$2x - 3y + 4z = 7$
$5x - 2y + 2z = 7$
$6x - 3y + 10z = 23$. **(07 Marks)**

b. Reduce the quadratic form : $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_12x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form by orthogonal transformation. **(07 Marks)**

c. Find the largest eigen value and corresponding eigen vector by Rayeligh's power method performing five iterations, with $x^{(0)} = [1, 1, 1]^T$ for $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **(06 Marks)**

10 a. Solve by LU decomposition method :
$10x + y + z = 12$
$2x + 10y + z = 13$
$2x + 2y + 10z = 14$. **(07 Marks)**

b. Diagonalze the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. Hence find A^4. **(07 Marks)**

c. Solve by Gauss Seidel iteration method :
$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$
Perform 3 iterations. **(06 Marks)**

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