

CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, June/July 2018 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

- 1 a. Find the n^{th} derivative of the $\sin^3 x \cos^2 x$. (06 Marks)
b. Find angle between the pair of curves $r = 6 \cos \theta$ and $r = 2(1 + \cos \theta)$. (05 Marks)
c. Show that for the curve $r(1 - \cos \theta) = 2a$ the radius of curvature is $\frac{2}{\sqrt{a}} r^{3/2}$. (05 Marks)

OR

- 2 a. Show that $\left(\frac{2\rho}{a}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ for the curve $y = \frac{ax}{a+x}$. (06 Marks)
b. Find the Pedal equation of the curve $r^m = a^m (\cos m\theta + \sin m\theta)$. (05 Marks)
c. If $y = \log(x + \sqrt{1+x^2})$ prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$. (05 Marks)

Module-2

- 3 a. Expand $\log(1+\cos x)$ by Maclaurin's series upto the term containing x^4 . (06 Marks)
b. Evaluate $\lim_{x \rightarrow \pi/2} (\sin x)^{\tan x}$. (05 Marks)
c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = u$ (05 Marks)

OR

- 4 a. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ show that $xu_x + yu_y = \sin 2u$. (06 Marks)
b. If $z = f(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$ (05 Marks)
c. Expand $\tan x$ in Taylor's series upto three in powers of $\left(x - \frac{\pi}{4}\right)$. (05 Marks)

Module-3

- 5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$, determine velocity and acceleration at $t = 1$. Also find the components of velocity and acceleration in the direction $2i + j + 2k$. (06 Marks)
b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$. (05 Marks)
c. Prove that $\text{Div}(\phi \vec{A}) = \phi(\text{div } \vec{A}) + \text{grad } \phi \cdot \vec{A}$ (05 Marks)

OR

- 6 a. Find the unit tangent vector and normal vector to the curve $\vec{r} = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$ at $x = \frac{1}{\sqrt{2}}$. (06 Marks)
- b. Find the $\text{curl}(\text{curl} \vec{A})$, where $\vec{A} = x^2 y \hat{i} - 2xz \hat{j} + 2yz \hat{k}$ at the point (1, 0, 2). (05 Marks)
- c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function of ϕ such that $\vec{F} = \nabla\phi$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x \, dx$. (06 Marks)
- b. Solve $xy(1+xy^2) \frac{dy}{dx} = 1$. (05 Marks)
- c. Show that the family of the curves $y^2 = 4a(x+a)$ is self orthogonal. (05 Marks)

OR

- 8 a. Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$. (05 Marks)
- b. Evaluate $\int_0^{\pi} \frac{\sin^4 \theta}{(1 + \cos \theta)^2} d\theta$. (05 Marks)
- c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C . (06 Marks)

Module-5

- 9 a. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, by using the power method by taking initial vector as $[1, 1, 1]^T$ (06 Marks)

- b. Find the rank of the matrix by reducing into the normal form, $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. (05 Marks)
- c. Solve the following system of equation by Gauss seidel method: $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. (05 Marks)

OR

- 10 a. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. (06 Marks)
- b. Solve by Gauss elimination method, $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$. (05 Marks)
- c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into the canonical form. (05 Marks)

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