ACHARYA INSTITUTE OF TECHNOLOGY
Bangalore - 560090

CBCS Scheme

USN | | | | | | | | | | |

15MATDIP31

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

1 a. Simplify $\frac{(\cos 3\theta - i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^5}{(\cos\theta + i\sin\theta)^3(\cos 2\theta - i\sin 2\theta)^4}$. **(06 Marks)**

b. Determine λ such that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar. **(05 Marks)**

c. Find sine angle of two vectors $4\hat{i} + 3\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + 2\hat{k}$. **(05 Marks)**

OR

2 a. Express $\frac{1}{2+i} - \frac{(1+i)^2}{3+i}$ in the form a + ib. **(06 Marks)**

b. Find modulus and amplitude of $1 + \cos\theta + i\sin\theta$. **(05 Marks)**

c. If $\vec{a} = 3\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} + 5\hat{j} + 10\hat{k}$ find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. **(05 Marks)**

Module-2

3 a. If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. With usual notation prove that $\tan\varphi = r\frac{d\theta}{dr}$. **(05 Marks)**

c. If $u = e^{ax+by} f(ax - by)$ prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$. **(05 Marks)**

OR

4 a. Find n^{th} derivative of $y = e^x \sin 4x \cos x$ **(06 Marks)**

b. Find pedal equation of $r = a(1 + \cos\theta)$. **(05 Marks)**

c. If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} \sin^5(x/2)dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(05 Marks)**

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

OR

6 a. Evaluate $\int_0^a \frac{x^3 dx}{\sqrt{a^2 - x^2}}$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx \, dy$. **(05 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$. **(05 Marks)**

Module-4

7 a. A particle moves along the curve c : $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where t denotes time. Find velocity and acceleration at $t = 2$. **(06 Marks)**

b. Find unit normal vector to surface $Q = x^2yz + 4xz^2$ at (1, -2, -1). **(05 Marks)**

c. Show that $\vec{f} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational. **(05 Marks)**

OR

8 a. A particle moves along the curve c : $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where 't' is the time. Find the components of velocity and acceleration at $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). **(05 Marks)**

c. If $\phi = 2x^3y^2z^4$ find div(gradϕ). **(05 Marks)**

Module-5

9 a. Solve : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. **(06 Marks)**

b. Solve : $x^2y \, dx - (x^3 + y^3) \, dy = 0$. **(05 Marks)**

c. Solve : $(y^3 - 3x^2y) \, dx - (x^3 - 3xy^2) \, dy = 0$. **(05 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)dx + xy \, dy = 0$. **(05 Marks)**

c. Solve : $\frac{dy}{dx} + y \cot x = \cos x$. **(05 Marks)**

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