USN ☐☐☐☐☐☐☐☐☐☐   **10MAT31**

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – III

Time: 3 hrs.   Max. Marks:100

Note: *Answer FIVE full questions, selecting*
at least TWO full questions from each part.

PART – A

1 a. Find the Fourier series of $f(x) = \begin{cases} \pi + 2x & \text{in} \quad -\pi \le x \le 0 \\ \pi - 2x & \text{in} \quad 0 \le x \le \pi \end{cases}$. **(06 Marks)**

   b. Obtain Fourier half range sine series of $f(x) = \begin{cases} \dfrac{1}{4} - x & ; \quad 0 < x < \dfrac{1}{2} \\ x - \dfrac{3}{4} & \quad \dfrac{1}{2} < x < 1 \end{cases}$. **(07 Marks)**

   c. Find the Fourier series of y upto second harmonics from the following table:

x:	0	2	4	6	8	10	12
y:	9	18.2	24.4	27.8	27.5	22	9

   (07 Marks)

2 a. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{for} \quad |x| \le 1 \\ 0 & \text{for} \quad \text{otherwise} \end{cases}$ and hence deduce that

   $\displaystyle\int_0^\infty \dfrac{x \cos x - \sin x}{x^3} . dx = \dfrac{-\pi}{4}$. **(07 Marks)**

   b. Find the inverse Fourier sine transform of $F_s(u) = \dfrac{1}{u} e^{-au}, a > 0$. **(06 Marks)**

   c. Find the Fourier cosine transform of $f(x) = \dfrac{1}{1 + x^2}$. **(07 Marks)**

3 a. Obtain the various possible solutions of the Laplace's equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

   b. Solve the heat equation $u_t = c^2 u_{xx}$ subject to the conditions, $u(0, t) = 0$, $u(10, t) = 0$ and
   $u(x, 0) = f(x)$ where $f(x) = \begin{cases} x & \text{in} \quad 0 \le x \le 5 \\ 10 - x & \text{in} \quad 5 \le x \le 10 \end{cases}$. **(06 Marks)**

   c. Obtain the D'Alembert's solution of the one dimensional wave equation. **(07 Marks)**

4 a. Fit a parabola of the form $y = ax^2 + bx + c$ to the following data:

x:	0	1	2	3	4	5
y:	1	3	7	13	21	31

   (06 Marks)

   b. Minimize: $z = 5x + 4y$ subject to the constraints $x + 2y \ge 10$, $x + y \ge 8$, $2x + y \ge 12$, $x \ge 0$, $y \ge 0$ by graphical method. **(07 Marks)**

   c. Maximize $z = 6x + 9y$ subject to the constraints $2x + 2y \le 12$, $x + 5y \le 44$, $3x + y \le 30$, $x \ge 0, y \ge 0$ by applying simplex method. **(07 Marks)**

PART – B

5 a. Using Regula-falsi method find the real root of tanx + tanhx = 0, which lies between 2 and 3 carryout three iterations. **(06 Marks)**

   b. Apply Gauss-Seidel method to solve equations $12x + y + z = 31$, $2x + 8y - z = 24$, $3x + 4y + 10z = 58$. Perform four iterations. **(07 Marks)**

   c. Using Rayleigh power method find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ use $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ as initial vector, carry out six iterations. **(07 Marks)**

6 a. From the following data, estimate the number of students who have scored less than 70 marks:

Marks:	0-20	20-40	40-60	60-80	80-100
No. of students:	41	62	65	50	17

(06 Marks)

   b. Use Lagrange's interpolation formula to fit a polynomial for the data:

x:	0	1	3	4
y:	-12	0	6	12

Hence estimate y at x = 2. **(07 Marks)**

   c. Evaluate, $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$ by using Simpsons 3/8th rule, taking six equal parts. **(07 Marks)**

7 a. Solve $\dfrac{\partial^2 u}{\partial t^2} = 4\dfrac{\partial^2 u}{\partial x^2}$ subject to u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 − x) by taking h = 1, k = 0.5 upto four steps. **(07 Marks)**

   b. Solve $\dfrac{\partial^2 u}{\partial x^2} = 32\dfrac{\partial u}{\partial t}$ subject to u(0, t) = 0, u(1, t) = t and u(x, 0) = 0 upto t = 5 by Bendre-Schmidt process taking $h = \dfrac{1}{4}$. **(07 Marks)**

   c. Solve $u_{xx} + u_{yy} = 0$ in the following square region with the boundary conditions as indicated in the figure: **(06 Marks)**
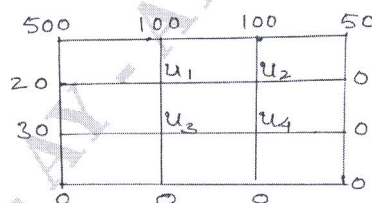

Fig.Q.7(c)

8 a. Find the Z-transforms of sinhnθ and coshnθ. **(06 Marks)**

   b. If $\bar{u}(z) = \dfrac{2z^2 + 5z + 14}{(z-1)^4}$. Find the values of u_0, u_1, u_2 and u_3. **(07 Marks)**

   c. Solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$ and $u_1 = 1$ by using z-transform. **(07 Marks)**

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