

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART - A

1. a. Using Taylor series method, solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ at the point $x = 0.2, 0.3$ consider up to 4th degree term. (06 Marks)
- b. Using Runge Kutta method of order 4, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2, 0.4$ by taking step length $h=0.2$. (07 Marks)
- c. Given $\frac{dy}{dx} = \frac{1}{2}xy, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, y(0.3) = 1.0228$. Compute y at $x = 0.4$ by Adams – Bash forth predictor – corrector method use corrector formula twice. (07 Marks)
2. a. Evaluate y and z at $x = 0.1$ from the Picard's second approximation to the solution of the following system of equations given by $y = 2$ and $z = 1$ at $x = 0$ initially $\frac{dy}{dx} = x + z$
 $\frac{dz}{dx} = x - y^2$. (06 Marks)
- b. Given $y'' = x^3(y + y')$ with the initial condition $y(0) = 1, y'(0) = 0.5$ compute $y(0.1)$ by taking $h = 0.1$ and using 4th order Runge Kutta method. (07 Marks)
- c. Applying Milne's method compute $y(0.4)$ Given that y satisfies the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$ and y and y' are governed by the following values
 $y(0) = 1, y(0.1) = 1.03995, y(0.2) = 1.138036$
 $y(0.3) = 1.29865, y'(0) = 0.1, y'(0.1) = 0.6955$
 $y'(0.2) = 1.258, y'(0.3) = 1.873$. (07 Marks)
3. a. Derive Cauchy Riemann Equation in Cartesian form. (06 Marks)
- b. Prove that for every analytic function $f(z) = u + iv$ the two families of curves $u(x,y) = C_1$ and $v(x,y) = C_2$ form an orthogonal system. (07 Marks)
- c. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analytic function of $z = x + iy$ find $f(z)$ in terms of $f(z)$. (07 Marks)
4. a. Find the bilinear transformation that maps the points $z = 0, i, \infty$ onto the points $w = 1, -i, -1$ respectively, find the invariant points. (06 Marks)
- b. Discuss the transformation $w = e^z$. (07 Marks)
- c. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where c is the circle $|z| = 3$. (07 Marks)

PART – B

- 5 a. Starting from Laplace differential equation. Obtain Bessel's differential equation as
 $xy'' + xy' + (x^2 - n^2)y = 0$ (08 Marks)
- b. If $x^3 + 2x^2 - x + 1 = a P_0(x) + b P_1(x) + c P_2(x) + d P_3(x)$ find the value of a, b, c, d. (06 Marks)
- c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{dy}{dx^n} (x^2 - 1)^n$ (06 Marks)
- 6 a. Define axioms of probability. Prove that,
 $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$ (06 Marks)
- b. A solar water heater manufactured by a company consists of two parts the heating panel and the insulated tank. It is found that 6% of the heaters produced by the company have defective heating panels and 8% have defective tank. Find the percentage of non defective heaters produced by the company. (07 Marks)
- c. A box contains 500 IC chips of which 100 are manufactured by company X and the rest by company Y. It is estimated that 10% of the chips made by company X and 5% made by company Y are defective. If a randomly selected chip is found to be defective find the probability that it came from company X. (07 Marks)
- 7 a. A random variables X takes the values -3, -1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) $p(-3 < x < 4)$ ii) $p(x \leq 2)$. (06 Marks)
- b. Find the mean and variance of binomial distribution. (07 Marks)
- c. In an examination 7% of students scores less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation of the marks are normally distribute, it is given that $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.4757) = 0.43$. (07 Marks)
- 8 a. Explain the following terms :
 i) Null hypothesis
 ii) Type I and Type II error
 iii) Confidence limits. (06 Marks)
- b. A coin is tossed 1000 times and it turn up head 540 times decide on the hypothesis that the coin is unbiased. (07 Marks)
- c. A certain stimulus administered to each of the 12 patients resulted is the following change is blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 can it be calculated that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 df 2.201.) (07 Marks)

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