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14ELD11

First Semester M.Tech. Degree Examination, June/July 2018
Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Construct a QR decomposition for :

$$A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 2\end{bmatrix}.$$ **(10 Marks)**

Working to six significant digits and show how round-off error can generate an incorrect Q matrix when the columns of A are linearly dependent.

b. Find the generalized inverse of :

$$\begin{bmatrix}-3 & 1\\ -2 & 1\\ -1 & 1\\ 0 & 1\\ 1 & 1\\ 2 & 1\\ 3 & 1\end{bmatrix}.$$ **(10 Marks)**

2 a. Construct a singular–value decomposition for the matrix :

$$\begin{bmatrix}2 & 2 & -2\\ 2 & 2 & -2\\ -2 & -2 & 6\end{bmatrix}.$$ **(10 Marks)**

b. Find the extrenals of the functional $y(x)=\int_{x_1}^{x_2}\frac{1+y^2}{y'^2}\,dx$. **(10 Marks)**

3 a. Show that the extermal of the isoperimetric problem $IY(x)=\int_{x_1}^{x_2} y'^2 dx$ subject to the condition $J[y(x)]=\int_{x_1}^{x_2} y^2 dx = K$ is a parabola. Determine the equation of the parabola passing through the points $P_1(1, 3)$ and $P_2(4, 24)$ and K = 36. **(10 Marks)**

b. For which range of the constant C is an extermal of the functional $\int_0^1\left(y'^2 - c^2y^2 - 2y\right)dx$, y(0) = 0, y(1) = 1 a minimizer. **(10 Marks)**

4 a. Solve the IBVP described by

PDE : $u_{tt} = c^2 u_{xx} + k$, $\quad 0 < x < 1,\ t > 0$

BCs : $u(0, t) = u_x(\ell, t) = 0$, $\quad t > 0$

ICs : $u(x, 0) = u_t(x, 0)$, $\quad 0 \le x \le \ell$. **(10 Marks)**

b. An infinitely long string having one end at $x = 0$ is initially at rest on the x–axis. The end $x = 0$ under goes a periodic transverse displacement described by $A_0 \sin\omega t$, $t > 0$. Find the displacement of any point on the string at any time t. **(10 Marks)**

5 a. Solve the heat conduction problem described by :

PDE : $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t}$, $\quad 0 < x < \infty, \quad t > 0$

BC : $u(0, t) = u_0$, $\quad t \ge 0$

IC : $u(x, 0) = 0$, $\quad 0 < x < \infty$

u and $\partial u/\partial x$ both tends to zero as $x \to \infty$. **(10 Marks)**

b. Determine the temperature distribution in the semi–infinite medium $x \ge 0$. When the end $x = 0$ is maintained at zero temperature and the initial temperature distribution is f(x). **(10 Marks)**

6 a. Solve the following bound any value problem in the half-plane $y > 0$, described by :

PDE : $u_{xx} + u_{yy} = 0$, $\quad -\infty < x < \infty, \quad y > 0$

BC : $u(x, 0) = f(x)$, $\quad -\infty < x < \infty$

u is bounded as $y \to \infty$ and $\dfrac{\partial u}{\partial x}$ both vanishes as $|x| \to \infty$. **(10 Marks)**

b. A uniform string of length L is stretched tightly between two fixed points at $x = 0$ and $x = L$. If it is displaced a small distance ε at a point $x = b$, $0 < b < L$, and released from rest at time $t = 0$, find an expression for the displacement at subsequent times. **(10 Marks)**

7 a. Solve by Big–M method :

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$. **(10 Marks)**

b. Use dual Simplex method to solve the LPP :

Maximize $z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \ge 1$

$x_1 + x_2 \le 7$

$x_1 + 2x_2 \ge 10$

$x_2 \le 3$

$x_1, x_2 \ge 0$. **(10 Marks)**

8 a. Solve the following problem by using the method of Lagrangian multiplier

Minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to the constraints $x_1 + x_2 + 3x_3 = 2$

$5x_1 + 2x_2 + x_3 = 5$

and $x_1, x_2 \ge 0$. **(10 Marks)**

b. Solve the following non-linear programming using Kuhn–Tucker conditions :

Maximize $z = x_1^2 - x_1x_2 - 2x_2^2$

Subject to $4x_1 + 2x_2 \le 24$

$5x_1 + 10x_2 \le 20$

$x_1, x_2 \ge 0$. **(10 Marks)**