

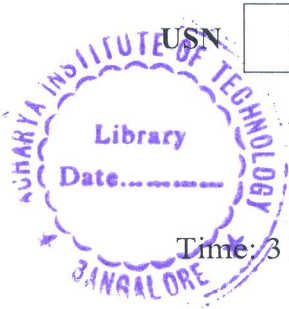
**Third Semester B.E. Degree Examination, June/July 2019**  
**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions.**

- 1 a. Express square root of  $1 - i$  in the form of  $x + iy$ . (07 Marks)
- b. Find the modulus and amplitude of the following and express each in polar form. (07 Marks)
  - (i)  $1 - i\sqrt{3}$
  - (ii)  $\frac{1-i}{1+i}$
- c. Expand  $\cos^6\theta$  in series of multiples of  $\theta$ . (06 Marks)
- 2 a. Find the  $n^{\text{th}}$  derivative of  $e^{ax} \cos(bx + c)$ . (06 Marks)
- b. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x+1)(x-2)}$ . (07 Marks)
- c. If  $y = \log(x + \sqrt{1+x^2})$ , prove that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y = 0$ . (07 Marks)
- 3 a. Find the angle between radius vector and the tangent of the curve  $r = a(1 + \cos \theta)$ . (06 Marks)
- b. Find the Taylor's series expansion of the function  $e^x$  about  $x = 1$ . (07 Marks)
- c. Obtain the Maclaurin's series expansion of the function  $\log_e(1+x)$  up to third degree terms. (07 Marks)
- 4 a. If  $\cos u = \frac{x+y}{\sqrt{x} + \sqrt{y}}$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \cot u$ . (06 Marks)
- b. If  $x = r \cos \theta$  and  $y = r \sin \theta$ , prove that  $JJ' = 1$ . (07 Marks)
- c. If  $x^y + y^x = c$ , where  $c$  is a constant, find  $\frac{dy}{dx}$ . (07 Marks)
- 5 a. Obtain the reduction formula  $I_n = \int \sin^n x \, dx$ , where  $n$  is a positive integer. (06 Marks)
- b. Evaluate:  $\int_0^1 \int_0^{\sqrt{x}} xy(x+y) \, dx \, dy$  (07 Marks)
- c. Evaluate:  $\int_0^1 \int_0^{1-z} \int_0^{1-z-y} (x+y+z) \, dx \, dy \, dz$  (07 Marks)
- 6 a. Prove the following:  $\beta(m, n) = \beta(n, m)$  (06 Marks)
- b. Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$  (07 Marks)
- c. Using Gamma function, evaluate the integral  $\int_0^1 \frac{1}{\sqrt{1-x^4}} \, dx$  (07 Marks)



- 7 a. Solve :  $(x + y + 1)^2 \frac{dy}{dx} = 1$  (06 Marks)
- b. Solve :  $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ . (07 Marks)
- c. Solve :  $(x^2 - xy + y^2)dx - xy dy = 0$  (07 Marks)
- 8 Solve the following second order O.D.Es.
- a.  $\frac{d^2y}{dx^2} + y = e^x$  (06 Marks)
- b.  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \cos^2 x$  (07 Marks)
- c.  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2(1+x)$ . (07 Marks)

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