CBCS SCHEME
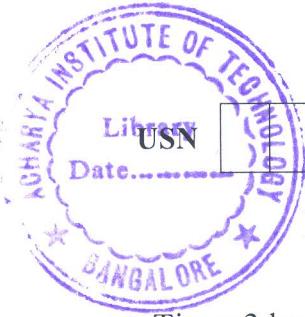
17MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, choosing
ONE full question from each module.**

Module-1

1 a. Find the nth derivative of $\sin 2x \, \sin 3x$. **(06 Marks)**

   b. Find the angle between the two curves $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$ **(07 Marks)**

   c. Find the radius of curvature for the curve $x^3 + y^3 = 3xy$ at $(3/2, 3/2)$. **(07 Marks)**

OR

2 a. If $y = \cos(m \log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$. **(06 Marks)**

   b. With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(07 Marks)**

   c. Find the pedal equation of the curve $r^m = a^m \cos m\theta$. **(07 Marks)**

Module-2

3 a. Find the Taylor's series of $\log(\cos x)$ in powers of $(x - \pi/3)$ upto fourth degrees terms. **(06 Marks)**

   b. If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ by using Euler's theorem. **(07 Marks)**

   c. If $u = \dfrac{yz}{x}, v = \dfrac{xz}{y}, w = \dfrac{xy}{z}$ then find $J = \dfrac{\partial(uvw)}{\partial(xyz)}$. **(07 Marks)**

OR

4 a. Evaluate $\displaystyle\lim_{x \to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$. **(06 Marks)**

   b. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{\underline{2}} - \dfrac{x^3}{\underline{3}} + \dfrac{x^4}{\underline{4}} - + - - - -$. **(07 Marks)**

   c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then prove that $\dfrac{1}{2}\dfrac{\partial u}{\partial x} + \dfrac{1}{3}\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the cuvre $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$. Find the components of velocity and acceleration in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 0$. **(06 Marks)**

   b. Find the constant a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational and find scalar potential function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

   c. Prove that $\mathrm{curl}(\phi\vec{A}) = \phi\,\mathrm{curl}\,\vec{A} + \mathrm{grad}\,\phi \times \vec{A}$. **(07 Marks)**

OR

6 a. Show that vector field $F = \dfrac{x\,\hat{i} + y\,\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. (06 Marks)

b. If $\vec{F} = (x + y + 1)\,\vec{i} + \vec{j} - (x + y)\,\vec{k}$ then prove that $\vec{F} = \text{curl}\,\vec{F} = 0$. (07 Marks)

c. Show that $\text{div}(\text{curl}\,\vec{A}) = 0$. (07 Marks)

Module-4

7 a. Obtain reduction formula for $\int \sin^n x\, dx\,(n > 0)$. (06 Marks)

b. Solve the differential equation $\dfrac{dy}{dx} + y\cot x = \cos x$. (07 Marks)

c. Find the orthogonal trajectory of the curve $r = a(1 + \sin\theta)$. (07 Marks)

OR

8 a. Evaluate $\displaystyle\int_0^{\pi/2} \sin^7\theta \cos^6\theta\, d\theta$. (06 Marks)

b. Solve the differential equation : $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. (07 Marks)

c. If the temperature of air is 30°C and the substance cools from 100°C to 70°C in 15 mins. Find when the temperature will be 40°C. (07 Marks)

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ by reducing to Echelon form. (06 Marks)

b. Find the largest eigen value and egien vector of the matrix : $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking initial vector as $[1\ 1\ 1]^T$ by using Rayleigh's power method. Carry out five iteration. (07 Marks)

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form, using orthogonal transformation. (07 Marks)

OR

10 a. Solve the system of equations
$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$
by using Gauss-Seidel method. Carry out three iterations. (06 Marks)

b. Diagonalise the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. (07 Marks)

c. Show that the transformation
$$y_1 = x_1 + 2x_2 + 5x_3$$
$$y_2 = 2x_1 + 4x_2 + 11x_3$$
$$y_3 = -x_2 + 2x_3$$
is regular. Write down inverse transformation. (07 Marks)

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