CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, June/July 2019
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. **(07 Marks)**

c. Find the value of a, b, c such that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational, also find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. Find the total work done in moving a particle in the force field $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2+1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. **(06 Marks)**

b. Using Green's theorem, evaluate

$$\int_C (xy + y^2)dx + x^2 dy,$$ where C is bounded by $y = x$ and $y = x^2$. **(07 Marks)**

c. Using Divergence theorem, evaluate $\int_S \vec{F}\,ds$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. **(07 Marks)**

Module-2

3 a. Solve $(D^2 - 3D + 2)y = 2x^2 + \sin 2x$. **(06 Marks)**

b. Solve $(D^2 + 1)y = \sec x$ by the method of variation of parameter. **(07 Marks)**

c. Solve $x^2y'' - 4xy' + 6y = \cos(2\log x)$ **(07 Marks)**

OR

4 a. Solve $(D^2 - 4D + 4)y = e^{2x} + \sin x$. **(06 Marks)**

b. Solve $(x+1)^2y'' + (x+1)y' + y = 2\sin[\log_e(x+1)]$ **(07 Marks)**

c. The current i and the charge q in a series containing an inductance L, capacitance C, emf E, satisfy the differential equation $L\frac{d^2q}{dt^2} + \frac{q}{C} = E$, Express q and i interms of 't' given that L, C, E are constants and the value of i and q are both zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z,\ x^2 + y^2 + z^2) = 0$ **(06 Marks)**

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ **(07 Marks)**

c. Derive one dimensional heat equation in the standard form as $\frac{\partial U}{\partial t} = C^2 \frac{\partial^2 U}{\partial x^2}$. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ such that $z = e^y$ where $x = 0$ and $\frac{\partial z}{\partial x} = 1$ when $x = 0$. **(06 Marks)**

b. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - \ell z)\frac{\partial z}{\partial y} = \ell y - mx$ **(07 Marks)**

c. Find all possible solutions of one dimensional wave equation $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Discuss the nature of the series $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$. **(06 Marks)**

b. With usual notation prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_3 + bP_2 cP_1 + dP_0$, find a, b, c and d using Legendre's polynomial. **(07 Marks)**

OR

8 a. Discuss the nature of the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{3.4} + \ldots\ldots\ldots\ldots$$ **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation in terms of $P_n(x)$

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$ **(07 Marks)**

c. Express $x^4 - 3x^2 + x$ interms of Legendre's polynomial. **(07 Marks)**

Module-5

9 a. Find the real root of the equation $x\sin x + \cos x = 0$ near $x = \pi$ using Newton-Raphson method. Carry out 3 iterations. **(06 Marks)**

b. From the following data, find the number of students who have obtained (i) less than 45 marks (ii) between 40 and 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 - 80
No. of Students	31	42	51	35	31

(07 Marks)

c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ using Simpson's $\frac{3}{8}^{th}$ rule by taking 7 ordinates. **(07 Marks)**

OR

10 a. Find the real root of the equation $x \log_{10} x = 1.2$ which lies between 2 and 3 using Regula-Falsi method. **(06 Marks)**

b. Using Lagrange's interpolation formula, find y at x = 4, for the given data:

x	0	1	2	5
y	2	3	12	147

(07 Marks)

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Weddle's rule by taking six equal parts. **(07 Marks)**

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