10MAT21

Second Semester B.E. Degree Examination, June/July 2019

Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The factor of $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$ is

A) $\left(P+\frac{y}{x}\right)\left(P-\frac{x}{y}\right)=0$ B) $\left(P-\frac{y}{x}\right)\left(P-\frac{x}{y}\right)=0$

C) $\left(P+\frac{y}{x}\right)(P+xy)=0$ D) $(Px+Py)(Px-Py)=0$

ii) The Integrating factor of the equation $y=2px+p^n$ is

A) p^3 B) $1/p^2$ C) $1/p$ D) p^2

iii) Which is the equation of the rectangular hyperbola

A) xz^2+c B) x^3y^2+c C) $xy=c$ D) xy^3+c

iv) Replace the differential equation $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, in this product of their slopes at each point of intersection is

A) +1 B) 2 C) 1/2 D) –1

b. Solve $y-2px=\tan^{-1}(xp^2)$. **(06 Marks)**

c. Solve $p=\sin(y-xp)$. Also find its singular solution. **(05 Marks)**

d. Find the curve for which the normal makes equal angles with the radius vector and the initial line. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) If roots of $m_1=\alpha+i\beta$, $m_2=\alpha-i\beta$, then the general solution is

A) $y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$ B) $y=e^{Ax}(\cos\alpha x+\sin\beta x)$

C) $y=(\cos\alpha x-\sin\beta x)$ D) $y=e^{x}(\cos x+i\sin x)$

ii) Which is the particular integral of $(D^2-5D+6)y=x$

A) $\frac{x}{5}+\frac{5}{18}$ B) $\frac{x}{6}+\frac{5}{36}$ C) $\frac{x^2}{2}+\frac{2}{7}$ D) $\frac{x}{3}+\frac{1}{2}$

iii) The number of auxillary roots of the $(D^3+2D^2+D)y=0$ are

A) 4 B) 2 C) 3 D) 1

iv) The complimentary function of the equation $(D^3+3D^2+3D+1)y=x^2$ is

A) $y=e^{-x}(c_1+c_2x+c_3x^2)$ B) $y=e^{x}(c_1x+c_2x^2+c_3)$

C) $y=c_1e^{-x}+c_2e^{-x}+c_3e^{-x})$ D) $y=(c_1+c_2x)e^{x}+c_3e^{-x}$

b. Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=\sin 3x\cos 2x$. **(06 Marks)**

c. Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=e^{2x}+x$ **(05 Marks)**

d. Solve the simultaneous equations

$\frac{dx}{dt}+5x-2y=t$, $\frac{dy}{dt}+2x+y=0$ being given $x=y=0$ when $t=0$. **(05 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) y_1 and y_2 are the solutions of $y'' + py' + qy = 0$, then which is the formula for finding Wronskian.

A) $W = \begin{vmatrix} y_1' & y_2' \\ y_1'' & y_2 \end{vmatrix}$ B) $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ C) $W = \begin{vmatrix} y_1 & y_2 \\ y_1'' & y_2'' \end{vmatrix}$ D) $W = \begin{vmatrix} y_2 & y_3 \\ y_2' & y_3'' \end{vmatrix}$

ii) Cauchy's homogeneous linear equation reduced to linear differential equation with constant coefficient by putting

A) $x = e^t$, $t = \log x$ B) $y = e^x$ C) $xy^2 = t$ D) $x + y = z$.

iii) Which is the general solution of the equation $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 0$

A) $y = c_1 \cos t + c_2 \sin t$ B) $y = c_1 \cos 2t + c_2 \sin 3t$
C) $y = c_1 \cos^2 t + c_2 t$ D) $y = (c_1 + c_2 t) \sin t$

iv) Which is the recurrence relation for series equation $\frac{d^2y}{dx^2} + xy = 0$

A) $a_n = \frac{a_{n+1}}{(n+1)^2}$ B) $a_{n+2} = \frac{-a_{n-1}}{(n+2)(n+1)}$ C) $a_{n-1} = \frac{a_n}{a_{n+1}}$ D) $a_n = \frac{a_{n+2}}{a_{n+1}}$

b. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = \log x$ **(06 Marks)**

c. Solve by method of variation of parameters $y'' + a^2 y = \sec ax$. **(05 Marks)**

d. Solve in series the equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$ **(05 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) Which partial derivative, denotes for S notation

A) $\frac{\partial z}{\partial x}$ B) $\frac{\partial^2 z}{\partial x \partial x^2}$ C) $\frac{\partial^2 z}{\partial x \partial y}$ D) $\frac{\partial^2 z}{\partial y^2}$

ii) Assume the trial solution for solving partial differential equation by separation of variables.

A) $Z = X_{(x)} Y_{(y)}$ B) $Z = X^2_{(x)} Y^2_{(y)}$ C) $Z = (XY)^2$ D) $Z = Y^2$

iii) The equation is of the form $P_p + Q_q = R$, the subsidiary equation is

A) $\frac{dx}{t} = \frac{dy}{s} = \frac{dz}{z}$ B) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ C) $\frac{dx}{x} = \frac{dy}{y} = dz$ D) $\frac{dx}{x} = \frac{dy}{y^2} = \frac{dz}{c}$

iv) The equation $\sqrt{p} + \sqrt{q} = 1$, then the desired solution is

A) $z = ax + (1-\sqrt{a})^2 y + c$ B) $z = ay + (1-\sqrt{b})^2 x + c$
C) $z = ax^2 + (1-\sqrt{a})^3 y + c$ D) $z = axy + c$

b. Form the partial differential equation by eliminating arbitrary function

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$ **(06 Marks)**

c. Solve by the method of separation of variables

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u, \text{ given that } u(0, y) = 2e^{5y}.$$ **(05 Marks)**

d. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following : (04 Marks)

i) The area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

A) $\pi^2 ab$ B) $\frac{\pi}{2}ab$ C) πab D) $\pi a^2 b^2$

ii) The value of $\int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)\,dz\,dy\,dx$ is

A) $abc(a^2+b^2+c^2)$ B) $\frac{abc}{3}(a^2+b^2+c^2)$ C) $a^2b^2c^2$ D) abc^2

iii) The definition of $\beta(m, n)$ is

A) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ B) $\int_0^1 x^{m-2}(1-x)^{n-2}$ C) $\int_0^1 x^m(1-x)^n$ D) $\int_0^1 x^{m+2}(1-x)^{m^2}$

iv) The coordinates of any point are (ρ, ϕ, z) and the transformation equations from Cartesian are

A) $x=\rho\cos\phi,\ y=\rho\sin\phi,\ z=z$ B) $x=\rho\sin^2\phi,\ y=\sin\phi,\ z=\rho$
C) $x=\rho\sin\phi,\ y=\cos^2\phi,\ z=3z$ D) $x=\rho\cos\phi,\ y=\sin\phi$

b. Using double integral find the area enclosed by the curve $r = a(1 + \cos\theta)$ and lying above the initial line. (06 Marks)

c. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$ (05 Marks)

d. Evaluate $\int_0^\infty e^{-x^2}dx$ (05 Marks)

6 a. Choose the correct answers for the following : (04 Marks)

i) If $\int_c F\cdot d\vec{r}=0$, then F is called

A) rotational B) solenoidal C) irrotational D) circulation.

ii) If $\vec{F}=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$, then the value of div $\vec{F}$ is

A) $2(x+y+z)$ B) $(x+y+z)$ C) (x^2+y+z) D) xyz

iii) Given the surface $x^2+y^2+z^2=a^2$, then the value of $\nabla\phi$ is

A) $2xi+2yj+2zk$ B) $2xi+2yj$ C) $3xi+2yj$ D) $(2x^2i+3yj+zk)$

iv) If $\vec{F}=xyi+yzj+zxk$, then the value of $\int_c \vec{F}\cdot d\vec{r}$ is

A) $\int_c xy\,dx+yz\,dy+zx\,dz$ B) $\int_c (xy\,dx+y^2z\,dy+z\,dz)$
C) $\int_c x^2i+yj+zk$ D) $\int_c xi+y^2j+zk$

b. Find the total work done by the force represented by $\vec{F}=3xyi-yj+2xzk$ in moving a particle round the circle $x^2+y^2=4$. (06 Marks)

c. Verify Stokes theorem for $\vec{F}=(x^2+y^2)i-2xyj$ taken round the rectangle bounded by the line $x=\pm a,\ y=0,\ y=b$. (05 Marks)

d. Using the divergence theorem find $\int_S F\cdot N\,ds$ where $\vec{F}=x^3i+y^3j+z^3k$ and S the surface of sphere $x^2+y^2+z^2=a^2$. (05 Marks)

7 a. Choose the correct answers for the following : **(04 Marks)**

i) The $L\{e^{at}t^n\}$ is

A) $\frac{1}{(s-a)^{n+1}}$ B) $\frac{n!}{(s-a)^{n+1}}$ C) $\frac{n^2}{s^{n+1}}$ D) $\frac{n!}{s^n}$

ii) If $L\{f(t)\} = f(s)$ then the value of $L\{\, t^n f(t)\,\}$ is

A) $(-1)^n \frac{d^n}{ds^n} f(s)$ B) $\frac{d^{n+1}}{ds^{n+1}}$ C) $\frac{d}{ds} f(s)$ D) $(-1)^n \frac{d^{n+1}}{ds^{n+1}}$

iii) If $L\{f(t) = f(s)\}$ then $L\{f(t-a)\, u(t-a)\}$ is

A) $e^t f(as)$ B) $e^{-as}\bar{f}(s)$ C) $e^{2s}\bar{f}(s)$ D) $e^{2as}\bar{f}(s)$

iv) If $L\{t^n\, \delta(t-a)\}$ is

A) $e^{-as} a^n$ B) $e^{ns} a$ C) $e^{3s} a^2$ D) $e^s a$

b. Find Laplace transform of the full wave rectifier $f(t) = E \sin wt,\ \ 0 < t < \pi/w$ having period π/w. **(06 Marks)**

c. Find the $L\{e^{3t} u(t-2)\}$ **(05 Marks)**

d. Using Laplace transform evaluate $\int_0^\infty e^{-t} \cdot t \cdot \sin^2 3t \cdot dt$ **(05 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) The inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ is

A) $\frac{t\cos at}{2a}$ B) $\frac{t\sin at}{2}$ C) $\frac{t\sin at}{2a}$ D) $t^2 \sin at$

ii) If $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\} = L^{-1}\left\{\frac{P}{s-1} + \frac{Qs+R}{s^2+1}\right\}$ then the values of P, Q and R are

A) P = 2, Q = –2, R = 1 B) P = 1, Q = 2, R = 2

C) P = 2, Q = 3, R = 1 D) P = 2, Q = 3, R = 2

iii) The inverse Laplace transform of $\tan^{-1}\left(\frac{2}{s}\right)$ is

A) $\frac{\sin 2t}{t}$ B) $\frac{\sin t}{3}$ C) $\frac{-\sin 2t}{t}$ D) $\cos 2t$

iv) The $L^{-1}\left\{\frac{s}{(s-a)^{n+1}}\right\}$ is

A) $\frac{e^{at}t^n}{n!}$ B) $\frac{e^t t^2}{n!}$ C) $e^{2t} t^3$ D) $e^t t^4$

b. Find the inverse Laplace transform of $\frac{2s-1}{s^2+2s+17}$. **(06 Marks)**

c. Using convolution theorem find inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. **(05 Marks)**

d. Solve the following initial value problem by using Laplace transform method $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-t}$, given $y(0) = 0$, $y'(0) = 0$. **(05 Marks)**

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