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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Find by Taylor's series method the value of y at x = 0.1 and x = 0.2 correct to four decimal places from $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1$. **(07 Marks)**

b. Using modified Euler's method find y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 taking h = 0.2 (use modified formula twice). **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1,\ y(1.1) = 1.233,\ y(1.2) = 1.548,\ y(1.3) = 1.979$. Evaluate y(1.4) by Adam's Bashforth method. **(06 Marks)**

2 a. Obtain the solution using fourth order Runge-Kutta method of the system of equations $\frac{dx}{dt} = 2x + y,\ \frac{dy}{dx} = x - 3y;\ t = 0,\ x = 0,\ y = 0.5$. Take h = 0.2. **(07 Marks)**

b. Obtain second approximation values of y and z to x = 0.1 using Picard's method, given that y(0) = 2, z(0) = 1 and $\frac{dy}{dx} = x + z, \frac{dz}{dx} = x - y^2$. **(07 Marks)**

c. Given y″ + xy′ + y = 0. Calculate y(0.4) using Milne's method by the following data :

x	0	0.1	0.2	0.3
y	1	0.995	0.9801	0.956
y′	0	–0.995	–0.196	–0.2867

(06 Marks)

3 a. Derive Cauchy-Riemann equations in polar form. **(06 Marks)**

b. If $\phi + i\Psi$ represents the complex potential of an electrostatic field where $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, find the complex potential as a function of z and hence determine ϕ. **(07 Marks)**

c. If f(z) is analytic, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^{|f(z)|^2} = 4|f'(z)|^2$. **(07 Marks)**

4 a. Find the bilinear transformation which maps the points z = 1, i, –1 into w = 0, 1, ∞. **(07 Marks)**

b. Find the image of the circles |z| = 1 and |z| = 2 under the mapping $w = z + \frac{1}{z}$. **(06 Marks)**

c. State and prove Cauchy's integral formula for the analytic function f(z) inside and on a simple closed curve. **(07 Marks)**

PART – B

5 a. Find the series solution of Bessel's differential equation leading to Bessel function. **(07 Marks)**

b. If α and β are two roots of $J_n(x) = 0$, then show that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Show that $x^4 - 3x^2 + x = \frac{8}{35}P_4(x) - \frac{10}{7}P_2(x) + P_1(x) = -\frac{4}{5}P_0(x)$. **(06 Marks)**

6 a. A problem in mathematics is given to three students A, B and C whose changes of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved? **(07 Marks)**

b. Give $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{5}$ and $P(A \cap B) = \frac{1}{20}$.

Find : i) $P(A/B)$ ii) $P(A/\overline{B})$ iii) $P(\overline{A}/\overline{B})$. **(06 Marks)**

c. State and proved Baye's theorem o conditional probability. **(07 Marks)**

7 a. A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of successes. **(07 Marks)**

b. A die is thrown 8 times. Find the probability that '3' falls,

i) Exactly 2 times
ii) Atleast once
iii) At the most 7 times. **(06 Marks)**

c. Define exponential distribution and obtain the mean and standard deviation of exponential distribution. **(07 Marks)**

8 a. Certain tubes manufactured by a company have mean life time of 800 hours and standard deviation of 60 hours. find the probability that a random sample of 16 tubes taken from the group will have a mean life time :

i) Between 770 hours and 830 hours
ii) Less than 785 hours
iii) More than 820 hours

(Given $\phi(2) = 0.4772$; $\phi(1) = 0.3413$; $\phi(1.33) = 0.4082$). **(07 Marks)**

b. A sample of 900 days was taken in a coastal town and it was found that on 100 days the weather was very hot. Obtain the probable limits of the percentage of very hot weather. **(06 Marks)**

c. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories ($\chi^2_{0.05} = 7.81$ for 3 d.f). **(07 Marks)**

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