CBCS SCHEME

USN

18EEE/ESE/EPS/EPE/EMS/ECD11

First Semester M.Tech. Degree Examination, June/July 2019
Mathematical Method in Control

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If $T : R^2 \to R^3$ is a Linear Transformation such that $T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$. Find X such that $T(x) = (-1, 4, 9)$. **(10 Marks)**

b. Define Vector space. If $T : R^n \to R^m$ is a Linear Transformation and $\{V_1, V_2, V_3\}$ be a Linearly dependent set in R^n, explain why the set $\{T(V_1), T(V_2), T(V_3)\}$ is Linearly dependent. **(10 Marks)**

OR

2 a. Find a basis for the column space of matrix and null space of the matrix A.

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix}$$ **(10 Marks)**

b. If $V_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $V_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $V_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ and $W = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

i) Is W in $\{V_1, V_2, V_3\}$? How many vectors are in $\{V_1, V_2, V_3\}$?
ii) How many vectors are in span $\{V_1, V_2, V_3\}$?
iii) Is W in the subspace spanned by $\{V_1, V_2, V_3\}$? Why? **(06 Marks)**

c. Show that W is in the subspace of R4 spanned by V_1, V_2, V_3 where

$$W = \begin{bmatrix} -9 \\ 7 \\ 4 \\ 8 \end{bmatrix} \quad V_1 = \begin{bmatrix} 7 \\ -4 \\ -2 \\ 9 \end{bmatrix} \quad V_2 = \begin{bmatrix} -4 \\ 5 \\ -1 \\ -7 \end{bmatrix} \quad V_3 = \begin{bmatrix} -9 \\ 4 \\ 4 \\ -7 \end{bmatrix}.$$ **(04 Marks)**

Module-2

3 a. Solve by Relaxation method, the equations

$9x - 2y + z = 50$
$x + 5y - 3z = 18$
$-2x + 2y + 7z = 19$. **(07 Marks)**

b. Find the Eigen values and Eigen vectors of the matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ **(06 Marks)**

c. Using Jacobi's method, find all the Eigen values and Eigen vectors of the matrix.

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$ **(07 Marks)**

OR

4 a. Solve the system of Equations by Croute's triangularisation method.

$x_1 + x_2 + x_3 = 1$
$4x_1 + 3x_2 - x_3 = 6$
$3x_1 + 5x_2 + 3x_3 = 4.$ **(07 Marks)**

b. Transform the matrix.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$ to tridiagonal form by Given's method. **(06 Marks)**

c. Determine the inverse of the matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

Using the Partition method, find the solution of the system of equations $x_1 + x_2 + x_3 = 1$, $4x_1 + 3x_2 + x_3 = 4$, $3x_1 + 5x_2 + 3x_3 = 6$. **(07 Marks)**

Module-3

5 a. If $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Then $\{X_1, X_2, X_3\}$ is clearly Linearly independent and thus is a basis for a subspace W of R^4 construct an orthogonal basis for W. **(10 Marks)**

b. Show that $\{V_1, V_2, V_3\}$ is an orthonormal basis of R3, where

$$V_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix} \quad V_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \quad V_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}.$$ **(06 Marks)**

c. Verify that $\{u_1, u_2\}$ is an orthogonal set and then find the orthogonal projection of y on to span $\{u_1, u_2\}$.

$$Y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} \quad u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$ **(04 Marks)**

OR

6 Find a singular value decomposition of A.

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$ **(20 Marks)**

Module-4

7 a. A random variable X has the following probability function for various values of x.

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find K ii) Evaluate $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$. Also find the probability distribution and the distribution function of X. **(08 Marks)**

b. Derive Mean and Standard deviation of the binomial distribution. **(06 Marks)**

c. X and Y are random variables having joint density function

$$f(x, y) = \begin{cases} 4xy & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Verify that i) $E(X+Y) = E(X) + E(Y)$ ii) $E(XY) = E(X) \cdot E(Y)$. **(06 Marks)**

OR

8 a. In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and S.D of 60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last far
i) More than 2100 hours ii) less than 1950 hours iii) between 1900 to 2100 hours.
(A(1.67) = 0.4525 , A(0.83) = 0.2967). **(06 Marks)**

b. The joint distribution of two random variables X and Y is as follows :

X \ Y	-4	2	7
1	-1/8	1/4	1/8
5	1/4	1/8	1/8

Compute the following : i) COV (X, Y) ii) ρ (X, Y). **(08 Marks)**

c. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains i) no defective fuses ii) 3 of more defective fuses. **(06 Marks)**

Module-5

9 a. If a Company employees n sales person's its gross sales in thousands of rupees may be regarded as Random variable having an Erlang distribution with $\lambda = \frac{1}{2}$ and $K = 80\sqrt{n}$. If the sales cost is Rs 8000 per sales person , how many sales person's should the company employ to maximize the expected profit. **(06 Marks)**

b. Obtain the characteristic function of the normal distribution. Deduce the first 4 central moments. **(10 Marks)**

c. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3/minute. Find the probability that during a time interval of 2 minutes.
i) Exactly 4 customers arrive ii) More than 4 customers arrive. **(04 Marks)**

OR

10 If Z and θ are independent Random variables, such that Z has a density function

$$f(z) = \begin{cases} 0 & \text{in } z < 0 \\ ze^{-z^2/2} & \text{in } z > 0 \end{cases}.$$

and θ is uniformly distributed $(0, 2\pi)$. Show that $\{X_t : -\infty < t < \infty\}$ is a Gaussian process if $X_t = z\cos(2\pi t + \theta)$ **(20 Marks)**

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