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10MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting atleast TWO questions from each part.

PART – A

- 1 a. Find the Fourier series of the function $f(x) = x$ over the interval $(-\pi, \pi)$. (06 Marks)
 b. Expand $f(x) = (x-1)^2$ as a half range cosine series in the interval $(0, 1)$. Hence show that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$. (07 Marks)
 c. For the function specified by the following table, find the Fourier series upto first harmonic.

θ°	0	60	120	180	240	300	360
$f(\theta)$	0.8	0.6	0.4	0.7	0.9	1.1	0.8

(07 Marks)

- 2 a. Find the Fourier transform of :
 $f(x) = 1 - x^2, |x| \leq 1$
 $= 0, |x| > 1$ (06 Marks)
 b. Find the inverse Fourier sine transform of $f_s(s) = \frac{e^{-as}}{s}$. (07 Marks)
 c. Obtain Fourier cosine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(07 Marks)

- 3 a. Obtain D'Alembert's solution of one dimensional wave equation :
 $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (06 Marks)
 b. Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < \ell$ under the conditions $u(0, t) = 0, u(\ell, t) = 0,$
 $u(x, 0) = \frac{kx}{\ell}$ where k is positive constant. (07 Marks)
 c. Obtain all possible solutions of Laplace equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 4 a. Fit a curve $y = ax + b$ by the least squares method to the data :

x	0	1	3	6	8
y	1	3	2	5	4

(06 Marks)

- b. Solve the following LPP by graphical method :

$$\text{Min } z = 20x_1 + 10x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1 \geq 0, x_2 \geq 0.$$

(07 Marks)

- c. Solve by Simplex Method :

$$\text{Max } z = 5x_1 + 3x_2$$

$$\text{Subject to } x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$x_1, x_2 \geq 0.$$

(07 Marks)

PART - B

- 5 a. Using Regula Falsi Method, find the root of the equation $xe^x = 2$ that lies between 0.8 and 0.9 correct to three decimal places. (06 Marks)

- b. Solve by Relaxation Method :

$$10x - 2y - 2z = 6$$

$$-x + 10y - 2z = 7$$

$$-x - y + 10z = 8.$$

(07 Marks)

- c. Find the largest eigenvalue and the corresponding eigenvector of the matrix :

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

by power method, taking initial vector $[1 \ 0 \ 0]'$. Perform 4 iterations.

(07 Marks)

- 6 a. Using Newton's backward interpolation formula find $f(13.2)$ given that :

x	10	11	12	13
f(x)	21	23	27	33

(06 Marks)

- b. Apply Lagrange's formula to find the value of x corresponding to $f(x) = 15$ from the data:

x	5	6	9	11
f(x)	12	13	14	16

(07 Marks)

- c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by using Simpson's $1/3^{\text{rd}}$ rule and taking six equal parts. Hence find the value of \log_c^2 . (07 Marks)

- 7 a. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the interior points of the square region shown in the Fig.Q7(a) (06 Marks)

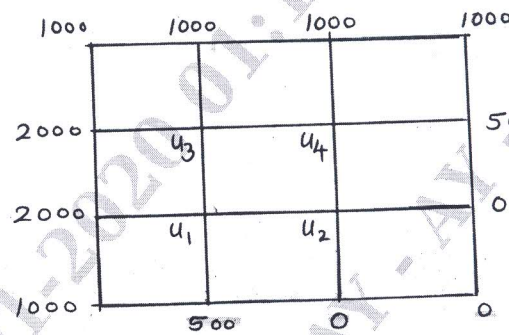


Fig.Q7(a)

- b. Obtain the numerical solution of heat equation $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$ under the condition $u(0, t) = u(4, t) = 0, t \geq 0$ and $u(x, 0) = x(4 - x), 0 < x < 4$ with $h = 1$ and $0 \leq t \leq 1, k = 1/4$. (07 Marks)
- c. By using the explicit three level formula solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0, t) = u(4, t) = 0, t \geq 0$, and initial conditions $u(x, 0) = x(4 - x), \frac{\partial u}{\partial t}(x, 0) = 0, 0 \leq x \leq 4$. Take $h = 1, k = 1/2$ and $0 < t \leq 2$. (07 Marks)
- 8 a. Prove that $Z(n^p) = -z \frac{d}{dz} Z(n^{p-1})$ where p is positive integer. (06 Marks)
- b. Find : i) $Z(n+1)$ ii) $Z[\sin(3n+5)]$. (07 Marks)
- c. Using z -transform solve the difference equation : $u_{n+2} - 3u_{n+1} + 2u_n = 0$, given $u_0 = 0, u_1 = -1$. (07 Marks)
