CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the Fourier expansion of the function f(x) = x over the interval $(-\pi, \pi)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ **(08 Marks)**

b. The following table gives the variations of a periodic current A over a certain period T:

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. **(08 Marks)**

OR

2 a. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. **(06 Marks)**

b. Represent the function

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2 & \text{for } \pi/2 < x < \pi \end{cases}$$

in a half range Fourier sine series. **(05 Marks)**

c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

x°	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(05 Marks)

Module-2

3 a. Find the complex Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$ Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ show that $u_0 = 0$ $u_1 = 0$ $u_2 = 2$ $u_3 = 11$. **(05 Marks)**

c. Obtain the Fourier cosine transform of the function

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0 & x > 4 \end{cases}$$ **(05 Marks)**

OR

4 a. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx \quad m > 0$. **(05 Marks)**

c. Solve by using Z-transforms $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$. **(05 Marks)**

Module-3

5 a. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at x = 6. **(06 Marks)**

x	1	2	3	4	5
y	10	12	13	16	19

b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(05 Marks)

c. Use Newton-Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$. Carryout the iterations upto four decimal places of accuracy. **(05 Marks)**

OR

6 a. Show that a real root of the equation $\tan x + \tanh x = 0$ lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. **(06 Marks)**

b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(05 Marks)

c. Fit a curve of the form $y = ae^{bx}$ for the data:

x	0	2	4
y	8.12	10	31.82

(05 Marks)

Module-4

7 a. From the following table find the number of students who have obtained:
i) Less than 45 marks
ii) Between 40 and 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

(06 Marks)

b. Construct the interpolating polgnomial for the data given below using Newton's general interpolation formula for divided differences and hence find y at x = 3.

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

(05 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule. Taking seven ordinates. Hence find $\log_e 2$. **(05 Marks)**

OR

8 a. Use Lagrange's interpolation formula to find f(4) given below. **(06 Marks)**

x	0	2	3	6
f(x)	-4	2	14	158

b. Use Simpson's 3/8th rule to evaluate $\int_1^4 e^{1/x} dx$. **(05 Marks)**

c. The area of a circle (A) corresponding to diameter (D) is given by

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(05 Marks)**

Module-5

9 a. Evaluate Green's theorem for $\phi_c (xy + y^2)\, dx + x^2 dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Find the extremal of the functional $\int_a^b (x^2 + y^2 + 2y^2 + 2xy) dx$. **(05 Marks)**

c. Varity Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ C is its boundary. **(05 Marks)**

OR

10 a. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y l}\right) = 0$. **(06 Marks)**

b. If $\vec{F} = 2xy\hat{i} + y^2 z\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. **(05 Marks)**

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