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MATDIP301

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express $\frac{(2+3i)^2}{(1+i)^2}$ in the form of complex number a + ib. **(06 Marks)**

b. Prove that $(1 + i)^4 + (1 - i)^4 = -8$. **(07 Marks)**

c. Find the cube root of $(\sqrt{3} - i)$. **(07 Marks)**

2 a. Find n^{th} derivative of $\sin(ax + b)$. **(06 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$. Show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. Find n^{th} derivative of $\log_e\left(\frac{2x+3}{2-3x}\right)^{\frac{1}{10}}$. **(07 Marks)**

3 a. Find the angle between the curves. $r = a(\sin\theta + \cos\theta)$ and $r = 2a\cos\theta$. **(06 Marks)**

b. Find the pedal equation for the curve $r^2 = a^2\sec(2\theta)$. **(07 Marks)**

c. Expand y = Log (cos x) using Maclaurin's series upto 4^{th} degree term. **(07 Marks)**

4 a. If $u = \sin^{-1}\left[\frac{x^3 + y^3 + z^3}{ax + by + cz}\right]$ show that $xu_x + yu_y + zu_z = 2\tan u$. **(06 Marks)**

b. If u = f(r, s, t) where $r = \frac{x}{y}$ $s = \frac{y}{z}$, $t = \frac{z}{x}$. Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. **(07 Marks)**

c. If $u = x + y + z$, $v = y + z$, $w = z$ find $\frac{\partial(uvw)}{\partial(xyz)}$. **(07 Marks)**

5 a. Obtain reduction formula for $\int \sin^n x \, dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate : $\int_0^1 x^9\sqrt{1-x^2}\,dx$. **(07 Marks)**

c. Evaluate : $\int_0^1\int_1^2 (x^2 + y^2)dxdy$. **(07 Marks)**

6 a. Evaluate : $\int_0^1\int_0^2\int_1^2 x^2yz \; dx\,dy\,dz$. **(06 Marks)**

b. Prove that $\beta(m,n) = \beta(n,m)$. **(07 Marks)**

c. Evaluate : $\int_0^2 \frac{x^2}{\sqrt{2-x}}dx$. **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = e^{-y}(e^x + x^2)$. **(06 Marks)**

b. Solve $(x^2 + y^2)dx = 2xy\,dy$. **(07 Marks)**

c. Solve $\frac{dx}{dy} = \frac{x}{y} + 2y^2$. **(07 Marks)**

8 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$. **(07 Marks)**

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