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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Mathematics - II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. If $[l_1, m_1, n_1]$ and $[l_2, m_2, n_2]$ be the direction cosines of two lines subtending an angle θ between them then prove that $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$. (06 Marks)
- b. Find the angle between two lines whose direction cosines satisfy the relations $l + m + n = 0$ and $2lm + 2nl - mn = 0$ (07 Marks)
- c. Find the co-ordinates of the foot of the perpendicular from $A(1,1,1)$ to the line joining $B(1,4,6)$ and $C(5,4,4)$. (07 Marks)
- 2 a. Find the equation of the plane which bisects the line joining $(3, 0, 5)$ and $(1, 2, -1)$ at right angles. (06 Marks)
- b. Show that the points $(2, 2, 0)$, $(4, 5, 1)$, $(3, 9, 4)$ and $(0, -1, -1)$ are coplanar. Find the equation of the plane containing them. (07 Marks)
- c. Find the shortest distance and the equations of the line of shortest distance between the lines:

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \text{ and } \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$$
 (07 Marks)
- 3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{b} = 5\hat{i} + 6\hat{j} + 4\hat{k}$ and $\vec{c} = 6\hat{i} + 4\hat{j} + 5\hat{k}$ form an isosceles triangle. (06 Marks)
- b. Prove that the points with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $\hat{j} + \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar. (07 Marks)
- c. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$ where t is the time t . Find the components of velocity and acceleration in the direction of the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 1$. (07 Marks)
- 4 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 - z^2 = 3$ at $(2, -1, 2)$. (06 Marks)
- b. Find the directional derivatives of the function $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2\hat{i} - \hat{j} - 2\hat{k}$ (07 Marks)
- c. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, -1, 1)$ where $\vec{F} = \nabla(xy^3z^2)$. (07 Marks)
- 5 a. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ then prove that,
 (i) $\nabla(r^n) = nr^{n-2}\vec{r}$ (ii) $\nabla \cdot (r^n \cdot \vec{r}) = (n+3)r^n$ (06 Marks)
- b. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
- c. Find the value of the constant 'a' such that $\vec{A} = y(ax^2 + z)\hat{i} + x(y^2 - z^2)\hat{j} + 2xy(z - xy)\hat{k}$ is Solenoidal. For this value of 'a' show that $\text{curl } \vec{A}$ is also solenoidal. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 a. Find the Laplace transform of, (i) $\sin 5t \cos 2t$ (ii) $(3t+2)^2$ (06 Marks)
- b. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. (07 Marks)
- c. Find the Laplace transform of $t^2 \sin at$. (07 Marks)
- 7 a. Find the inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$. (06 Marks)
- b. Find $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$. (07 Marks)
- c. Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$. (07 Marks)
- 8 a. Using convolution theorem find the Laplace transform of $\frac{1}{s(s^2+a^2)}$. (10 Marks)
- b. Solve the differential equation, $y'' + 5y' + 6y = 5e^{2x}$ under the condition $y(0) = 2$, $y'(0) = 1$ using Laplace transform. (10 Marks)

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