FREE VIBRATIONAL ANALYSIS OF SQUARE PLATE UNDER DIFFERENT BOUNDARY CONDITIONS USING ANSYS AND MATLAB

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Abstract: Vibration is a mechanical phenomenon where a body/member oscillates about an equilibrium point. Vibration of plates is a special case problem in mechanical field. Plate is a very basic and simple structure to study the structural behaviour, mechanical properties, etc. A surface of any plane undergoes huge amount of vibration which might cause instability to the structure and may result in a failure. This surface is divided into square plates and analysis of such square plate is studied in this paper. This square plate needs to be constrained in such a way that the whole structure will not only be stable but also eliminates the unwanted resonant frequency. To eliminate the state of resonance, it is very important to identify the natural frequency of a plate under different boundary conditions. This paper mainly focuses on identifying natural frequency of a plate constrained in various manners. This paper also compares ANSYS results with the theoretical results using MATLAB. As a result of this comparison, a slight variation in values are obtained but no significant trend is observed. The results obtained by MATLAB are obtained from theoretical formulations. For this study, Modal analysis is considered to find the natural frequencies of an aluminum isotropic square plate. For MATLAB calculations, Navier solution is used to determine the natural frequency for different modes (m, n). The effect of different boundary conditions on the natural frequencies and mode shapes are also investigated.

***Index terms*—Natural frequency, modal analysis, ANSYS and MATLAB.**

I. INTRODUCTION

Systems with a finite number of degrees of freedom are called as discrete system, whereas those with infinite number of degrees of freedom are called as continuous system. In continuous systems, mass is distributed over a large area instead of lumped mass due to which it has infinite mass points. The governing equation of plates are simpler than those for general 3D objects because the thickness dimension of the plate is much smaller than the other two dimensions. This suggest a 2D plate theory will give excellent approximation as similar to 3D plate like structure [3]. To find the natural frequency of such continuous system, Modal analysis is considered. For this analysis, it is assumed that the plate is homogeneous and obeys Hooke's law. Modal analysis is the study of dynamic properties of the system in the frequency domain, where there is no external force acting on the structure after giving an initial disturbance. The governing equation for isotropic rectangular plate shown in figure 1 by neglecting the thermal loads as given by equation 1.

II. MATHEMATICAL MODELLING

2.1 Theoretical formulations

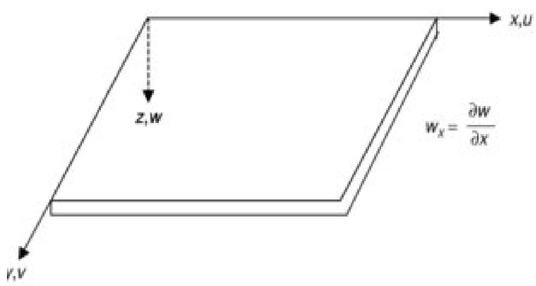


Figure 1. Displacement in plate

There are several plate theories that have been developed to describe the motion of plates. The most commonly used are the Kirchhoff-Love theory and the Mindlin-Reissner theory. The solution to governing equations predicted by these theories will give the behaviour of plate like objects for both free and forced conditions.

The governing equation of motion for linear bending of elastic plates in the absence of thermal load and without elastic foundation is given by

$$D_{11}\frac{\partial^4 w_o}{\partial x^4} + 2D_{12}\frac{\partial^4 w_o}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_o}{\partial y^4} + I_o\frac{\partial^2 w_o}{\partial t^2} - I_2\left(\frac{\partial^4 w_o}{\partial t^2 \partial x^2} + \frac{\partial^4 w_o}{\partial t^2 \partial y^2}\right) = q \quad (2.1)$$

The Navier method can be used to determine the solution for simply supported plate and the resulting ordinary differential equation is solved analytically or by numerical method.

Consider a plate with all sides simply supported the boundary conditions can be expressed in terms of *w* as

$$w = 0, \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0 \quad (2.2)$$

in the Navier solution, the mode shape of the plate for mode (m, n) is given by

$$D_{11}\alpha_m^4 + 2\widehat{D}_{12}\alpha_m^2\beta_n^2 + D_{22}\beta_n^4 - \omega^2[I_o + (\alpha_m^2 + \alpha_m^2)I_2] = 0 \quad (2.3)$$

Where,

w= displacement in z-axis.
$\alpha_m = m\pi/a$ and $\beta_m = n\pi/b$.
m & *n* are mode shape along *x* and *y*.
a= length of plate.
b= width of plate.
ω= natural frequency.
I_0 = Mass moment of inertia ($I_o = \rho h$)
I_2 = rotary inertia.

By neglecting rotary inertia (I_2), then the natural frequency of orthotropic plate is

$$\omega_{mn}^2 = \frac{\pi^2}{\rho h b^4}\left[D_{11}m^4\left(\frac{b}{a}\right)^4 + 2(D_{12} + 2D_{66})m^2n^2\left(\frac{b}{a}\right)^2 + D_{22}n^4\right]$$

For an isotropic plate

$$\omega_{mn} = \frac{\pi^2}{b^2}\sqrt{\frac{D}{\rho h}}\left(m^2\frac{b^2}{a^2} + n^2\right) \quad (2.4)$$

Where,

D = stiffness co-efficient = $\sqrt{\frac{Eh^3}{12(1-\mu^2)}}$

For this study a sample plate is considered with 70 GPa as modulus of elasticity, density of 2700 kg/m^3 and Poison's ratio of 0.3. The dimensions of this plate are as shown in figure 2.

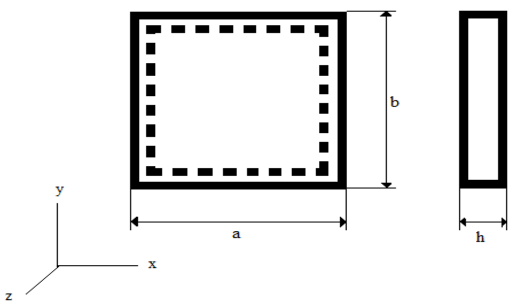


Figure 2. Free body diagram of simply supported plate dimension.

For square plate length (a) = width (b), then the equation 2 will be reduced to

$$\omega_{mn} = \frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}\,(m^2 + n^2) \quad rad/s \quad (2.5)$$

Above equation is used for finding frequencies only for simply supported plates for other boundary conditions cannot get the solution by this method. So, by using non dimensional frequency equation for further solution is given by

$$\omega_{mn} = \frac{\lambda^2 ij}{a^2}\sqrt{\frac{D}{\rho h}} \quad (2.6)$$

Where, λ^2_{ij} a non-dimensional parameter is depending on boundary condition. [4]

For present work, four boundary conditions are considered to determine the natural frequencies and mode shapes for an isotropic square plate as shown in Figure 3.

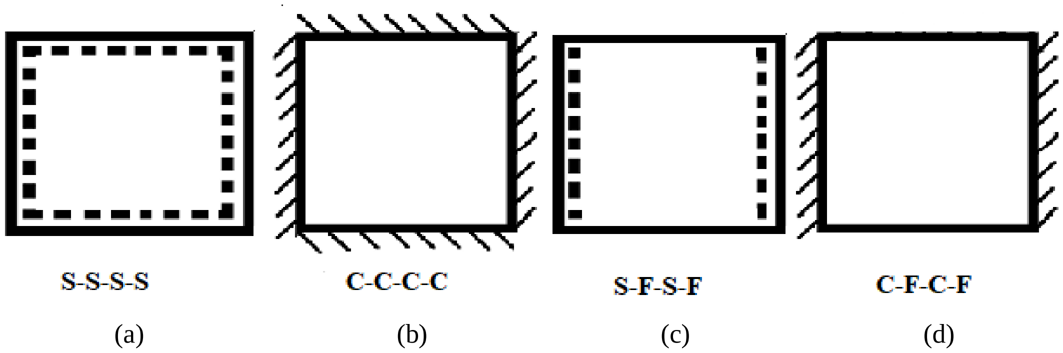


Figure 3. Different boundary conditions taken for plate (a) S-S-S-S - All sides are simply supported (b) C-C-C-C - All sides are clamped (c) S-F-S-F - Two sides Simply supported -Two sides are free (d) C-F-C-F - Two sides are clamped- Two sides are free.

2.2 Numerical analysis

A Convergence test is conducted to determine the size of elements in finite element modelling. The numerical solution must converge or must tend to the exact solution of the problem. In finite element modelling a finer mesh typically results in a more accurate solution, however as a mesh is made finer the computation time increases to get the results. This refers to the smallness of the elements required in a modal to ensure that the results of an analysis are not affected by changing the size of the mesh. The sequence of approximate solution will converge to the exact solution if the interpolation model satisfies the following convergence requirements.

Plates with given dimensions are modelled using ANSYS (19.2). The element should consider as shell 181 type is preferred because it is suitable for thin to moderately thick plates and shell structures. It is a rectangular element in which each element will have 4 nodes at the corners and each node will have 6 degrees of Freedom. To reduce the computational time, an optimum mesh size is considered. At the end of convergence test, we should get accurate values for natural frequencies at different modes. These values are compared with values obtained by analytical methods.

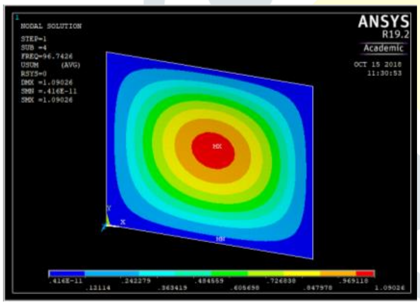
III. RESULTS AND DISCUSSION

3.1 Convergence study

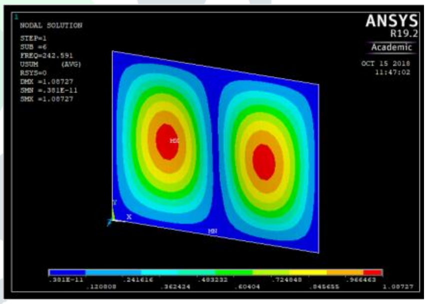
For this plate with all sides simply supported, the convergence test is as shown in the table 2.1. It is very clear that no significant changes were observed (before decimal point) in the values after mesh size of 2500 elements. Therefore, optimum mesh size is 2500 elements which can give accurate answers.

Table 2.1. Convergence test for simply supported square plate

ANSYS result			
No. of Mesh Elements	Natural frequency (Hz) at Mode (1, 1)	Natural frequency (Hz) at Mode (2, 1) (1,2)	Natural frequency (Hz) at Mode (2, 2)
100	97.615	250.934	400.838
400	96.906	243.906	389.85
900	96.74	242.59	387.72
1600	96.661	242.096	386.885
2500	96.608	241.84	386.434
3600	96.568	241.685	386.144



(a)



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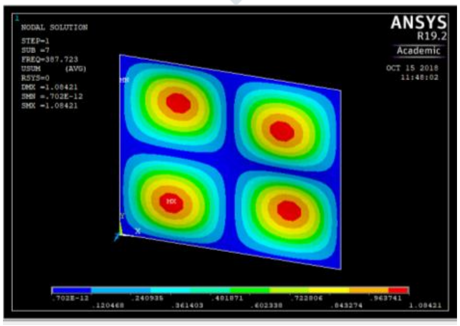


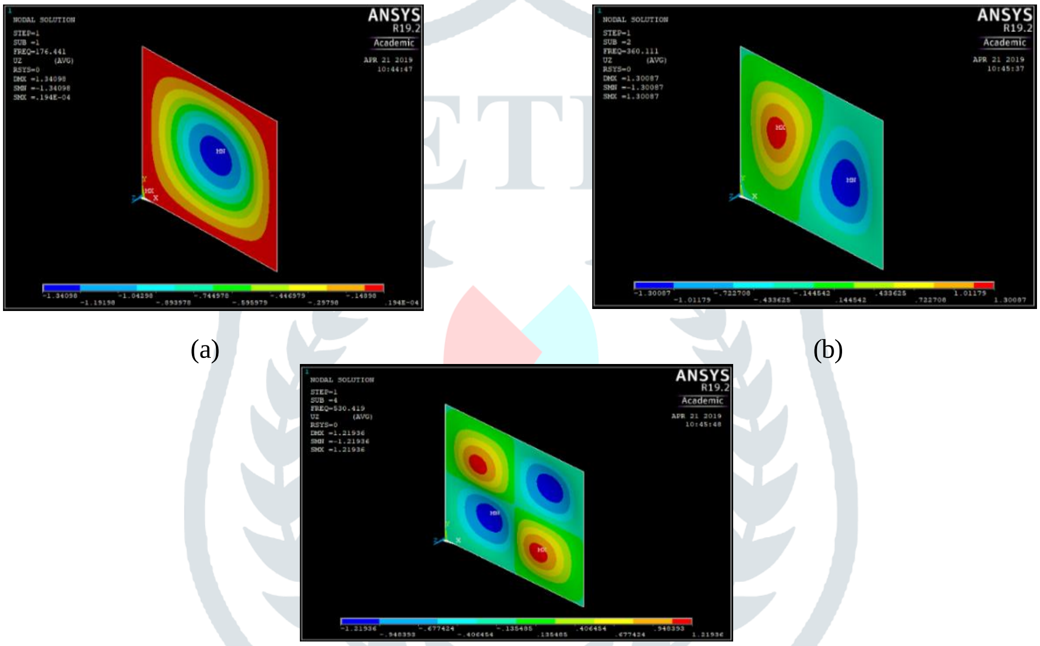
(c)

Figure 4. Natural frequency at (a) mode 1,1 (b) mode 1,2 (c) mode 2,2 for simply supported plate.

After convergence test, the same mesh size of 2500 elements are applied on all samples with different boundary conditions and the results obtained are tabulated in Table 2.2

Table 2.2. Natural frequency for plates with different boundary conditions at different modes

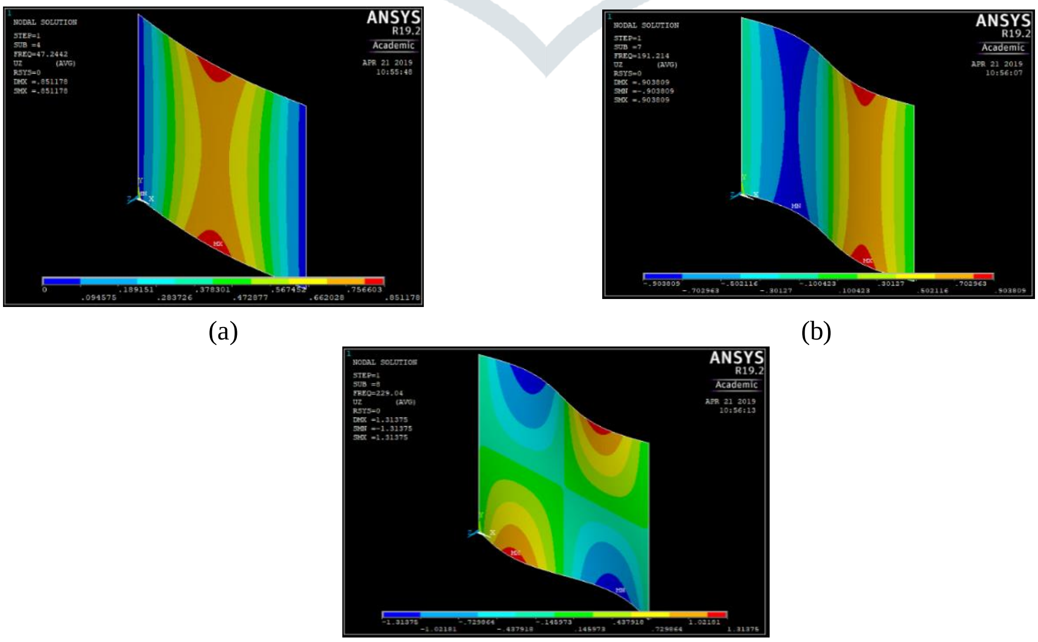
SL NO	PLATE TYPE	Mode shapes	ANSYS results (Hz)
1	S-S-S-S	Mode 1,1	96.608
		Mode 2,1	241.84
		Mode 2,2	386.434
2	C-C-C-C	Mode 1,1	176.527
		Mode 2,1	360.063
		Mode 2,2	531.023
3	S-F-S-F	Mode 1,1	47.238
		Mode 2,1	191.010
		Mode 2,2	229.232
4	C-F-C-F	Mode 1,1	74.967
		Mode 2,1	243.907
		Mode 2,2	277.685



(a) (b)

(c)

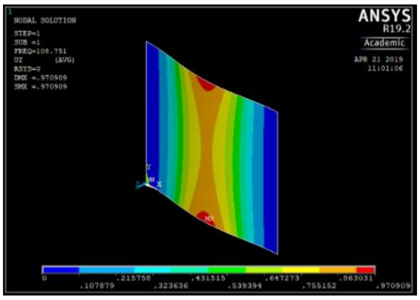
Figure 5. Natural frequency at (a) mode 1,1 (b) mode 2,1 (c) mode 2,2 for C-C-C-C plate



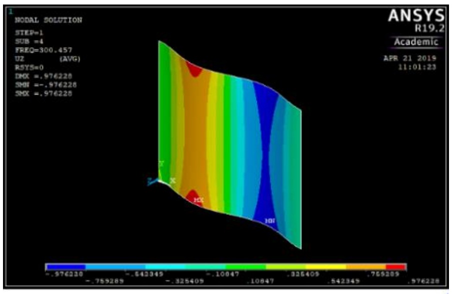
(a) (b)

(c)

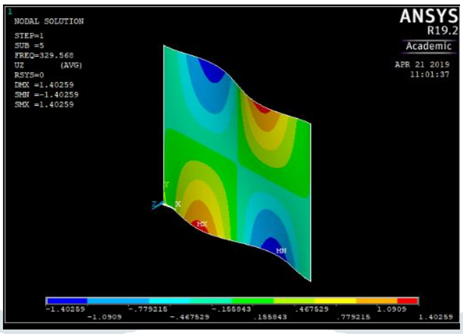
Figure 6. Natural frequency at (a) mode 1,1 (b) mode 2,1 (c) mode 2,2 for S-F-S-F plate



(a)



(b)



(c)

Figure 7 Natural frequency at (a) mode 1,1 (b) mode 2,1 (c) mode 2,2 for C-F-C-F plate

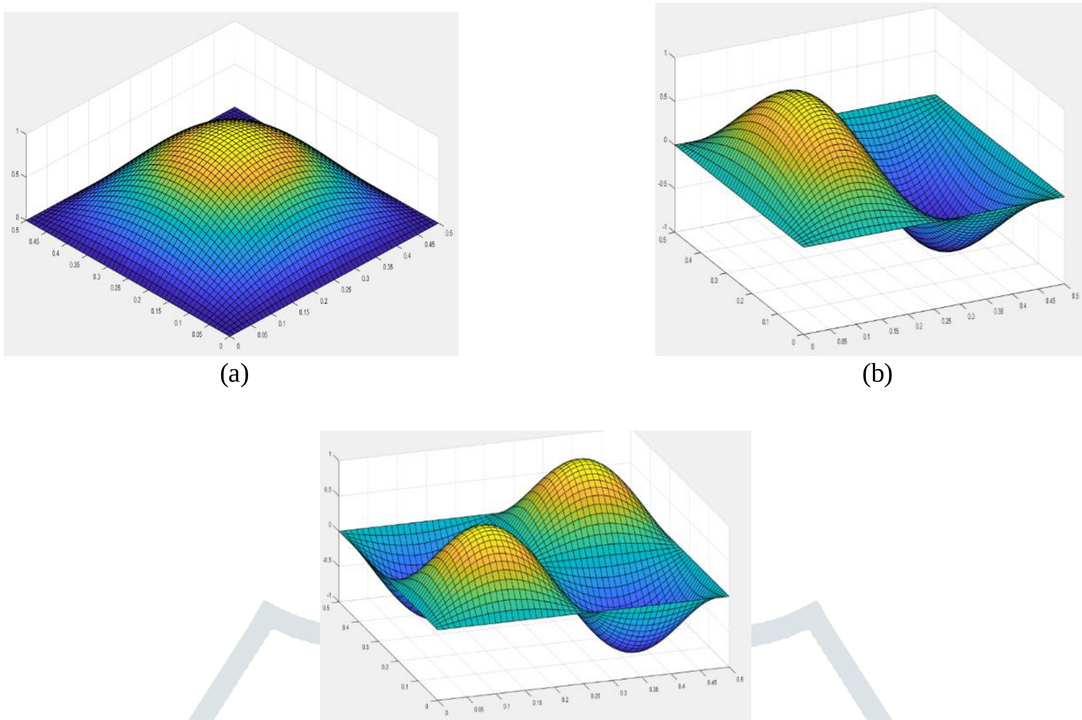
3.2 Theoretical solution obtained from MATLAB

Theoretical solutions for free vibration analysis of square plates in the form of natural frequencies and mode shapes are obtain by equation (2.4) by code developed in computer software MATLAB. The natural frequency results obtained by MATLAB are tabulated in Table 3.1 and mode shape (1, 1), (2,1), (2,2) are shown in Figure 8. For all sides simply supported and clamped boundary conditions mode 2,1 and 1,2 are the same value because of symmetrical nature.

Table 3.1. Natural frequency for plates in different boundary condition

SL NO	PLATE TYPE	Mode shapes	$\lambda^2{}_{ij}$	Theoretical Natural frequency (Hz)
1	S-S-S-S	Mode 1,1	19.739	96.812
		Mode 2,1& 1,2	49.348	242.033
		Mode 2,2	78.956	387.249
2	C-C-C-C	Mode 1,1	35.992	176.527
		Mode 2,1&1,2	73.413	360.063
		Mode 2,2	108.27	530.023
3	S-F-S-F	Mode 1,1	9.6314	47.238
		Mode 1,2	16.134	79.132
		Mode 2,1	38.945	191.011
		Mode 2,2	46.738	229.232
4	C-F-C-F	Mode 1,1	22.272	109.235
		Mode 1,2	26.529	130.114
		Mode 2,1	61.466	301.467
		Mode 2,2	67.549	331.302

The graphical representations of these results for different modes are shown in fig. 8.



(a) (b)

(c)

Figure 8. MATLAB results for (a) mode 1, 1 (b) mode 2, 1 (c) mode 2, 2

IV. OBSERVATION

The results obtained by ANSYS for isotropic plate with all boundary conditions are compared with theoretical solutions obtained by MATLAB is mentioned in table 4.1.

Table 4.1 comparison between ANSYS results and Theoretical results

SL NO	PLATE TYPE	Mode shapes	ANSYS Results (Hz)	Theoretical results (Hz)	Difference (Hz)
1	S-S-S-S	Mode 1,1	96.608	96.741	0.133
		Mode 2,1	241.84	242.591	0.751
		Mode 2,2	386.434	387.723	1.289
2	C-C-C-C	Mode 1,1	176.527	176.441	0.086
		Mode 2,1	360.063	360.110	0.047
		Mode 2,2	531.023	530.418	0.605
3	S-F-S-F	Mode 1,1	47.238	47.244	0.006
		Mode 2,1	191.010	191.213	0.203
		Mode 2,2	229.232	229.039	0.193
4	C-F-C-F	Mode 1,1	74.967	74.527	0.44
		Mode 2,1	243.907	242.822	1.085
		Mode 2,2	277.685	275.964	1.721

It is observed from Table 4.1 that maximum difference between theoretical and ANSYS values for natural frequency were found at higher mode shape in both directions. Therefore, it can be concluded that as number of mode shapes increases in both x and y direction, higher amount of difference in natural frequency obtained by Ansys and theoretical calculations were observed.
Further studies can be made for different plate shapes like rectangular, with different boundary conditions by adding stiffeners. Also, an attempt can be made for studies on parameters like varying plate thickness, width and length.

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