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Third Semester B.E. Degree Examination, Aug./Sept. 2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Let A, B, C, $\subseteq R^2$ where $A = \{(x, y)/y = 2x + 1\}$, $B = \{(x, y)/y = 3x\}$ and $C = \{(x, y)/x - y = 7\}$. Determine each of the following :
i) $A \cap B$ ii) $B \cap C$ iii) $\overline{A} \cup \overline{C}$ iv) $\overline{B} \cup \overline{C}$. **(05 Marks)**

b. Using Venn diagram, prove that, for any sets A, B, C $\overline{(A \cap B) \cup C} = (\overline{A} \cup \overline{B}) \cap \overline{C}$. **(05 Marks)**

c. At a high school science fair, 34 students received awards for scientific projects. Fourteen awards were given for projects in biology, 13 in chemistry, and 21 in physics. If three students received awards in all three subject areas, how many received awards for exactly i) one subject area? ii) two subject areas? **(05 Marks)**

d. Prove that the open interval (0, 1) is not a countable set. **(05 Marks)**

2 a. Define tautology. By constructing truth table, verify that $\{[(p \vee q) \to r] \wedge (\neg p)\} \to (q \to r)$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences without using truth tables.
(i) $[\neg p \wedge (q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$
(ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$
(iii) $\neg(p \downarrow q) \Leftrightarrow \neg p \uparrow \neg q$ **(07 Marks)**

c. Define valid argument.
Write the following argument in symbolic form and then establish the validity of the argument:
If the band could not play rock music or the refreshments were not delivered on time, then the New year's party would have been canceled and Alicia would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music. **(08 Marks)**

3 a. Consider the following open statements with the set of all real numbers as the universe.
$p(x) : x \geq 0$ $q(x) : x^2 \geq 0$, $r(x) : x^2 - 3x - 4 = 0$
$s(x) : x^2 - 3 > 0$.
Determine the truth values of the following statements. If a statement is false, provide a counter example or explanation.
(i) $\exists x(p(x) \wedge q(x))$
(ii) $\forall x(p(x) \to q(x))$
(iii) $\forall x(q(x) \to s(x))$
(iv) $\forall x(r(x) \vee s(x))$
(v) $\exists x(p(x) \wedge r(x))$
(vi) $\forall x(r(x) \to p(x))$ **(06 Marks)**

b. For a prescribed universe and any open statements p(x), q(x) in the variable x, prove that $\exists x[p(x) \vee q(x)] \Leftrightarrow [\exists x\, p(x) \vee \exists x\, q(x)]$ **(06 Marks)**

c. Prove that the following argument is valid

$$\exists x\,(p(x) \wedge q(x)) \rightarrow \forall y\,(r(y) \rightarrow s(y))$$
$$\exists y(r(y) \wedge \neg s(y))$$
$$\overline{\therefore \forall x(p(x) \rightarrow \neg q(x))}$$

(08 Marks)

4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's using the alternative form of the principle of mathematical induction. **(06 Marks)**

b. A sequence $\{a_n\}$ is defined recursively by $a_n = 3a_{n-1} - 2a_{n-2}$, $a_1 = 5$, $a_2 = 3$. Find a_n in explicit form. **(07 Marks)**

c. For $n \geq 0$ let F_n denote the n^{th} Fibonacci number. Prove that $F_0 + F_1 + \ldots. + F_n = F_{n+2} - 1$. **(07 Marks)**

PART – B

5 a. If $f : Z \rightarrow R$ is defined by $f(x) = x^2 + 5$, find $f^{-1}(\{6\})$, $f^{-1}([6, 10])$, $f^{-1}([5, \infty])$. **(05 Marks)**

b. Show that every set of seven distinct integers includes two integers x and y such that at least one of x + y or x – y is divisible by 10. **(05 Marks)**

c. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one – to – one and onto. **(05 Marks)**

d. Let $f : Z \rightarrow N$ be defined by

$$f(x) = \begin{cases} 2x - 1 & \text{if } x > 0 \\ -2x & \text{if } x \leq 0 \end{cases}$$

Prove that f is a bijection and find f^{-1}. **(05 Marks)**

6 a. For A = {1, 2, 3, 4} let R = {(1, 1), (1, 2), (2, 3), (3, 3), (3, 4)} be a relation on A. Find R^2, R^3 and R^4 and draw their digraphs. **(05 Marks)**

b. Let A = {1, 2, 3, 6, 9, 18} and define R on A by xRy if x|y. Draw the Hasse diagram for the poset (A, R). **(06 Marks)**

c. Consider the Hasse diagram of a poset (A,R) shown in Fig Q6(c)

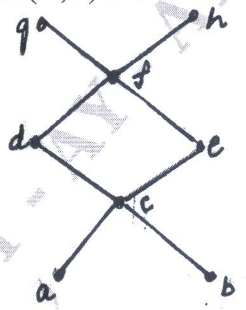

Fig Q6(c)

if B = {c, d, e) find (if they exist)

(i) all upper bounds of B
(ii) all lower bounds of B
(iii) the least upper bound of B
(iv) the greatest lower bound of B **(04 Marks)**

d. On Z, define the relation R by xRy if and only if 4/x – y. verify that R is an equivalence relation. Find the equivalence class of any integer i with respect to R. **(05 Marks)**

7 a. Let $G = \{q \in Q / q \neq -1\}$. Define the binary operation 0 on G by $x \bullet y = x + y + xy$. Prove that (G, 0) is an abelian group. **(05 Marks)**

b. Let G be a group and let $J = \{x \in G / xy = yx \text{ for all } y \in G\}$. Prove that J is a subgroup of G. **(05 Marks)**

c. Prove that every subgroup of a cyclic group is cyclic. **(05 Marks)**

d. Let (G, 0), (H, $*$) be groups with respective identities e_G, e_H. If f : G $\rightarrow$ H is a homomorphism then prove that

(i) $f(e_G) = e_H$

(ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G$

(iii) f(S) is a subgroup of H for each subgroup S and G. **(05 Marks)**

8 a. An encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by the generation matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(i) Determine all the code words. What can be said about the error – detection capability of this code? What about its error – correction capability?

(ii) Find the associated parity – check matrix H

(iii) Use H to decode the received words: 11101, 11011. **(10 Marks)**

b. Define a ring and integral domain. Let R be a commutative ring with unity. Prove that R is an integral domain if and only if for all a, b, c, $\in R$ where $a \neq z$ (additive identify) $ab = ac \Rightarrow b = c$. **(10 Marks)**

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