CBCS SCHEME

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18MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation, prove that for the curve $r = f(\theta)$, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(06 Marks)**

b. Find the radius of curvature at any point P(x, y) on the parabola $y^2 = 4ax$. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

OR

2 a. Find the pedal equation of the curve $\frac{2a}{r} = 1 - \cos\theta$. **(06 Marks)**

b. Find the radius of curvature of the tractrix $x = a[\cos t + \log\tan(t/2)], y = a\sin t$. **(06 Marks)**

c. Show that the angle between the pair of curves: $r = 6\cos\theta$ and $r = 2(1 + \cos\theta)$ is $\pi/6$. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that
$\sqrt{1+\sin 2x} = 1 + x - x^2/2! - x^3/3! + x^4/4!$ **(06 Marks)**

b. Evaluate: i) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$ ii) $\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$ **(07 Marks)**

c. Examine the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for is extreme values. **(07 Marks)**

OR

4 a. If $U = f(x - y, y - z, z - x)$ show that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If $u = x\cos y\cos z$, $v = x\cos y\sin z$, $w = x\sin y$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -x^2\cos y$. **(07 Marks)**

c. Find the volume of the largest rectangular parallelopiped that can be inscribed in the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dz\,dy\,dx$ **(06 Marks)**

b. Find the volume of the solid bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$ **(07 Marks)**

c. Show that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Change the order of Integration and hence evaluate
$$\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$$ (06 Marks)

b. Find the centre of gravity of the curve $r = a(1 + \cos\theta)$. (07 Marks)

c. Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \cdot \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$ (07 Marks)

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. (06 Marks)

b. Find the orthogonal trajectories of the family of cardioides $r = a(1 + \cos\theta)$ (07 Marks)

c. Solve: $[4x^3y^2 + y\cos(xy)]dx + [2x^4y + x\cos(xy)]dy = 0$ (07 Marks)

OR

8 a. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t. (06 Marks)

b. Solve : $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$ (07 Marks)

c. Solve : $x^2p^2 + xp - (y^2 + y) = 0$, where $p = \frac{dy}{dx}$. (07 Marks)

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by applying elementary row operations. (06 Marks)

b. Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh's power method taking the initial Eigen vector as $[1, 1, 1]^T$. (07 Marks)

c. Apply Gauss-Jordon method to solve the system of equations:
$2x + 5y + 7z = 52, \quad 2x + y - z = 0, \quad x + y + z = 9.$ (07 Marks)

OR

10 a. Test for consistency and solve:
$5x_1 + x_2 + 3x_3 = 20, 2x_1 + 5x_2 + 2x_3 = 18, 3x_1 + 2x_2 + x_3 = 14$ (06 Marks)

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. (07 Marks)

c. Solve the system of equations
$5x + 2y + z = 12$
$x + 4y + 2z = 15$
$x + 2y + 5z = 20$
Using Gauss-Siedel method [carry out 4 iterations]. (07 Marks)

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