



Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

17MAT31

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Fourier series to represent the periodic function $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. (08 Marks)

- b. The following table gives the variations of periodic current over a period.

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amp.	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Expand A as a Fourier series upto first harmonic. Obtain the amplitude of the first harmonic. (06 Marks)

- c. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. (06 Marks)

OR

- 2 a. Find the Fourier series of $f(x) = 2x - x^2$ in $(0, 3)$. (08 Marks)
 b. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier series expansion of y as given in the following table: (06 Marks)

x:	0	1	2	3	4	5	6
y:	9	18	24	28	26	20	9

- c. Obtain the half-range sine series for the function,

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases} \quad (06 \text{ Marks})$$

Module-2

- 3 a. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \leq a \\ 0, & |x| > a \end{cases}$. Hence deduce that

$$\int_0^{\infty} \frac{(\sin x - x \cos x)}{x^3} \cos \frac{x}{2} dx = \frac{3\pi}{16} \quad (08 \text{ Marks})$$

- b. Find the Z-transform of,
 (i) $\cos n\theta$ and (ii) $\cosh n\theta$ (06 Marks)
 c. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0 = y_1$, using z-transforms technique. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

- 4 a. Find the Fourier cosine transform of e^{-ax} . Hence evaluate $\int_0^{\infty} \frac{\cos \lambda x}{x^2 + a^2} dx$ (08 Marks)
- b. Find the Z-transform of,
 (i) $(n+1)^2$ (ii) $\sin(3n+5)$ (06 Marks)
- c. Find the inverse Z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (06 Marks)

Module-3

- 5 a. Find the two regression lines and hence the correlation coefficient between x and y from the data. (08 Marks)
- | | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y | 10 | 12 | 16 | 28 | 28 | 36 | 41 | 49 | 40 | 50 |
- b. Fit a second degree parabola to the following data: (06 Marks)
- | | | | | | |
|---|---|-----|-----|-----|-----|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |
- c. Using Newton-Raphson method find the root of $x \sin x + \cos x = 0$ near $x = \pi$ corrected to 4 decimal places. (06 Marks)

OR

- 6 a. Two variables x and y have the regression lines $3x + 2y = 26$ and $6x + y = 31$. Find the mean values of x and y and the correlation coefficient between them. (08 Marks)
- b. Fit a curve of the form, $y = ae^{bx}$ to the following data: (06 Marks)
- | | | | | | | |
|----|----|----|----|----|----|----|
| x: | 5 | 15 | 20 | 30 | 35 | 40 |
| y: | 10 | 14 | 25 | 40 | 50 | 62 |
- c. Using Regula-Falsi method find the root of $xe^x = \cos x$ in the interval (0, 1) carrying out four iterations. (06 Marks)

Module-4

- 7 a. Using Newton's forward and backward interpolation formulae, find $f(1)$ and $f(10)$ from the following table: (08 Marks)
- | | | | | | | | |
|------|-----|-----|------|------|------|------|------|
| x | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| f(x) | 4.8 | 8.4 | 14.5 | 23.6 | 36.2 | 52.8 | 73.9 |
- b. Given that $f(5) = 150$, $f(7) = 392$, $f(11) = 1452$, $f(13) = 2366$, $f(17) = 5202$. Using Newton's divided difference formulae find $f(9)$. (06 Marks)
- c. Using Simpson's $\frac{1}{3}$ rule evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates. (06 Marks)

OR

- 8 a. Using Newton's Backward difference interpolation formula find $f(105)$ from, (08 Marks)
- | | | | | | |
|------|------|------|------|------|------|
| x | 80 | 85 | 90 | 95 | 100 |
| f(x) | 5026 | 5674 | 6362 | 7088 | 7854 |
- b. If $f(1) = -3$, $f(3) = 9$, $f(4) = 30$, $f(6) = 132$ find Lagrange's interpolation polynomial that takes the same value as $f(x)$ at the given point. (06 Marks)
- c. Evaluate $\int_4^{5.2} \log_e x dx$ by Simpson's $\frac{3}{8}$ rule with $h = 0.1$. (06 Marks)

Module-5

- 9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2 dy$ where C is bounded by $y = x$ and $y = x^2$.
(08 Marks)
- b. Using Gauss divergence theorem evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$,
where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallel piped $0 \leq x \leq a$,
 $0 \leq y \leq b$ and $0 \leq z \leq c$.
(06 Marks)
- c. With usual notations derive Euler's equation, $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.
(06 Marks)

OR

- 10 a. If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, evaluate $\oint_C \vec{F} \cdot d\vec{r}$ along the curve C in the xy-plane, $y = x^3$
from (1, 1) to (2, 8).
(08 Marks)
- b. Find the extremals of the functional with $y(0) = 0$ and $y(1) = 1$.
(06 Marks)
- c. Show that Geodesics on a plane arc straight lines.
(06 Marks)
