USN

MATDIP401

Fourth Semester B.E. Degree Examination, Aug./Sept.2020

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angles between any two diagonals of a cube. **(06 Marks)**

b. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are the direction cosines of two lines then angle θ between the lines is $\theta = \cos^{-1}(\ell_1 \ell_2 + m_1 m_2 + n_1 n_2)$. **(07 Marks)**

c. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, show that : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$. **(07 Marks)**

2 a. Find the equation of the plane through (1, –2, 2), (–3, 1, –2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

b. Find the equation of the line passing through the points (1, 2, –1) and (3, –1, 2). At what point does it meet the yz – plane. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect. Find the point of intersection and the equation of the plane in which they lie. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} + 4\hat{k}$ from a right–angle triangle. **(06 Marks)**

b. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. **(07 Marks)**

c. Find the constant a so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} - a\hat{j} - 5\hat{k}$ are coplanar. **(07 Marks)**

4 a. If $\frac{d\vec{A}}{dt} = \vec{W} \times \vec{A}$, $\frac{d\vec{B}}{dt} = \vec{W} \times \vec{B}$, show that $\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{W} \times (\vec{A} \times \vec{B})$. **(06 Marks)**

b. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $2\hat{i} - 3\hat{j} - 6\hat{k}$. **(07 Marks)**

c. Find the angle between the surfaces $x^2yz + 3xz^2 = 5$ and $x^2y^3 = 2$ at (1, –2, –1). **(07 Marks)**

5 a. Find unit vector normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2). **(06 Marks)**

b. Prove that $\text{curl}(\phi\vec{A}) = (\text{grad}\phi) \times \vec{A} + \phi\,\text{curl}\,\vec{A}$. **(07 Marks)**

c. Prove that $\nabla^2(r)^n = n(n+1)r^{n-2}$, where $r = |x\,i + y\,j + z\,k|$. **(07 Marks)**

6 a. Find Laplace transform of coshat. (06 Marks)

b. If $f(t)=\begin{cases} 2t & \text{for } 0\le t\le 5 \\ 1 & \text{for } t>5 \end{cases}$, find L[f(t)]. (07 Marks)

c. Find $L\left[\dfrac{\cos 2t-\cos 3t}{t}\right]$. (07 Marks)

7 a. By using the convolution theorem find the inverse Laplace transforms of $\dfrac{1}{s^2(s+5)}$. (06 Marks)

b. Find $L^{-1}\left[\dfrac{(3s+7)}{s^2+2s-3}\right]$. (07 Marks)

c. Find the inverse Laplace transform of $\log\left(1+\dfrac{a^2}{s^2}\right)$. (07 Marks)

8 a. Using Laplace transform solve :

$\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+4y=e^{-t}, y(0)=0=y'(0)$. (10 Marks)

b. Solve the system of equations by the method of Laplace transform

$(D-2)x+3y=0,\ 2x+(D-1)y=0$

Where $D=\dfrac{d}{dt}$, given that x = 8, y = 3 at t = 0. (10 Marks)

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