



CBCS SCHEME

15MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2021

Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the real and imaginary parts of $\frac{2+i}{3-i}$ and express in the form of $x + iy$. (05 Marks)
- b. Reduce $1 - \cos \alpha + j \sin \alpha$ to the modulus amplitude form $[r(\cos \theta + i \sin \theta)]$ by finding r and θ . (06 Marks)
- c. If $\vec{a} = 4i + 3j + k$ and $\vec{b} = 2i - j + 2k$ find the unit vector perpendicular to both the vectors \vec{a} and \vec{b} . Hence show that $\sin \theta = \frac{\sqrt{185}}{3\sqrt{26}}$ where ' θ ' is angle between \vec{a} and \vec{b} . (05 Marks)

OR

- 2 a. Find the modulus and amplitude of $\frac{3+i}{1+i}$. (05 Marks)
- b. Find 'a' such that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. (06 Marks)
- c. Show that for any three vectors $\vec{a}, \vec{b}, \vec{c}$ $[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}] = [\vec{a}, \vec{b}, \vec{c}]^2$. (05 Marks)

Module-2

- 3 a. Find the n^{th} derivative of $\sin(5x) \cos(2x)$. (05 Marks)
- b. If $y = a \cos(\log x) + b \sin(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)
- c. If $u = \sin^{-1} \frac{x+y}{\sqrt{x}-\sqrt{y}}$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$. (05 Marks)

OR

- 4 a. Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4 . (05 Marks)
- b. Give $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$, $y = t^2$ find $\frac{du}{dt}$ as a function of t . (06 Marks)
- c. If $x = r \cos \theta$, $y = r \sin \theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$. (05 Marks)

Module-3

- 5 a. State reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ and evaluate $\int_0^{\pi/2} \sin^9 x \, dx$. (05 Marks)
- b. Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$. (06 Marks)
- c. Evaluate: $\int_0^1 \int_0^2 \int_0^2 x^2 yz \, dx \, dy \, dz$. (05 Marks)

OR

- 6 a. Evaluate : $\int_0^{\pi} \sin^4 x \cos^6 x \, dx$. (05 Marks)
- b. Evaluate : $\int_0^5 \int_0^{x^2} y(x^2 + y^2) \, dx \, dy$. (06 Marks)
- c. Evaluate : $\int_0^1 \int_0^2 \int_0^2 x^3 y^2 z^3 \, dx \, dy \, dz$. (05 Marks)

Module-4

- 7 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the velocity and acceleration at time $t = 1$. (05 Marks)
- b. Find the unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. (06 Marks)
- c. What is solenoid vector field? Demonstrate that vector \vec{F} given by $\vec{F} = 3y^2z^3\mathbf{i} + 8x^2\sin(z)\mathbf{j} + (x+y)\mathbf{k}$ is solenoidal. (05 Marks)

OR

- 8 a. Find $\text{div } F$ and $\text{Curl } F$ if $\vec{F} = (3x^2 - 3yz)\mathbf{i} + (3y^2 - 3xz)\mathbf{j} + (3z^2 - 3xy)\mathbf{k}$. (05 Marks)
- b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (06 Marks)
- c. Show that the fluid motion $\vec{V} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$ is irrotational. (05 Marks)

Module-5

- 9 Find the solution of :
- a. $(x^2 + 2e^x)dx + (\cos y - y^2)dy = 0$. (05 Marks)
- b. $\frac{dy}{dx} = \frac{y/x}{1 + y/x}$. (06 Marks)
- c. $(x^2 - ay)dx + (y^2 - ax)dy = 0$. (05 Marks)

OR

- 10 a. Find the solution of : $\frac{dy}{dx} = \frac{x^3}{y^3}$. (05 Marks)
- b. $(x^2y^3 + \sin x)dx + (x^3y^2 + \cos y)dy = 0$. (06 Marks)
- c. $\cos y \frac{dy}{dx} + \sin y = 1$. (06 Marks)

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