CBCS SCHEME

USN | | | | | | | | | |

15MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $x^2e^x\cos x$. **(06 Marks)**

b. Find the angle (ϕ) between the radius vector and tangent of the curve $r = a(1+\sin\theta)$. Also and determine the slope of the curve $a+\theta = \frac{\pi}{2}$. **(05 Marks)**

c. Obtain the angle of intersection of the polar curves $r = a\log\theta$; $r = \frac{a}{\log\theta}$. **(05 Marks)**

OR

2 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of the polar curve $r^n = a^n\cos n\theta$. **(05 Marks)**

c. Find the radius of curvature at any point 't' of the curve, $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$. **(05 Marks)**

Module-2

3 a. Expand $y = \log x$ in powers of $(x-1)$ upto fourth degree term and hence evaluate $\log(1.1)$. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. **(05 Marks)**

c. If $u - \log(x^3+y^3+z^3-3xy)$, prove the following:

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ **(05 Marks)**

OR

4 a. Prove that, using MaClaurin's series, $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots$ **(06 Marks)**

b. If $u = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{4}\sin 2u$ **(05 Marks)**

c. If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$, $w = \frac{zx}{y}$, find $J\left(\frac{u,v,w}{x,y,z}\right)$. **(05 Marks)**

Module-3

5 a. Find the constants 'a' and 'b' such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. Also find s scalar potential ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) along the vector $\hat{A} = 2\hat{i} - \hat{j} - 2\hat{k}$. **(05 Marks)**

c. A particle moves along the curve $\vec{r} = 2t\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$. Find the components of velocity and acceleration in the direction of the vector $\vec{A} = i - 3\hat{j} + 2\hat{k}$ at $t = 2$. **(05 Marks)**

OR

6 a. For any scalar field ϕ and any vector field $\vec{A}$, prove that $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla\phi) \times \vec{A}$. **(06 Marks)**

b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find div($\vec{F}$) and curl($\vec{F}$) at the point (1, 2, 3). **(05 Marks)**

c. Find the angle between the tangents to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at $t = 1$ and $t = 2$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **(05 Marks)**

c. Find the orthogonal trajectories of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$. **(05 Marks)**

OR

8 a. Evaluate $\int_0^a x\sqrt{ax - x^2}\,dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. **(05 Marks)**

c. The temperature of a body drops from 100°C to 75°C in 10 minutes when the surrounding air is at 20°C. what will be its temperature after half an hour? When will be the temperature be 25°C? **(05 Marks)**

Module-5

9 a. Show that the linear transformation : $y_1 = 2x_1 + x_2 + x_3$; $y_2 = x_1 + x_2 + 2x_3$; $y_3 = x_1 - 2x_3$ is regular. Also, determine the inverse transformation. **(06 Marks)**

b. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

using Rayleigh's power method. Choose $[1, 0, 0]^T$ as the initial vector and perform five iterations. **(05 Marks)**

c. Solve the following system of equation by Gauss elimination method:

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

(05 Marks)

OR

10 a. Employ the Gauss-Seidal method to solve the following system:

$$9x - y + 2z = 9$$
$$x + 10y - 2z = 15$$
$$-2x + 2y + 13z = 17$$

Choose (1, 1, 1) as the starting solution and carry out four iterations. **(06 Marks)**

b. Reduce the following matrix to diagonal form:

$$A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

(05 Marks)

c. Obtain the canonical form of the quadratic form $3x^2 + 2y^2 - z^2 + 12yz + 8zx - 4xy$. **(05 Marks)**

* * * * *