USN

First Semester B.E. Degree Examination, Jan./Feb.2021

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting ONE full question from each module.***

Module – 1

1 a. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

b. Find the Pedal equation for the curve $r^n = a^n \cos n\theta$. **(06 Marks)**

c. Show that the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ is $4a\cos\left(\frac{\theta}{2}\right)$. **(07 Marks)**

2 a. Find the n^{th} derivative of $\cos 2x \cos 3x \cos 5x$. **(07 Marks)**

b. Find the angle between the radius vector and the tangent and also find the slope of the tangent for the curve $\frac{2a}{r} = 1 - \cos\theta$ at $\theta = \frac{2\pi}{3}$. **(07 Marks)**

c. Derive an expression to find radius of curvature in pedal form. **(06 Marks)**

Module – 2

3 a. Obtain Maclaurin's series for log(secx) upto the term containing x^6. **(07 Marks)**

b. If u is a homogeneous function of degree 'n' in x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(06 Marks)**

c. If $u = x+y+z$, $v = y+z$, $w = z$ then find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

4 a. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$. **(07 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. **(06 Marks)**

c. Find the extreme values of $\sin x + \sin y + \sin(x+y)$. **(07 Marks)**

Module – 3

5 a. A particle moves along the curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$, where t denotes time. Find the components of its acceleration at $t = 2$ along the tangent and normal. **(07 Marks)**

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$ $(\alpha \geq 0)$ using differentiation under the internal sign where α is the parameter. **(06 Marks)**

c. Apply the general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

6 a. Find the angle between the tangents to the curve $\vec{r} = t^2 i + 2tj - t^3 k$ at the points $t = \pm 1$. **(07 Marks)**

b. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(06 Marks)**

c. Show that $\text{curl}(\text{grad}\phi) = \vec{0}$. **(07 Marks)**

Module – 4

7 a. Obtain the reduction formula for $\int \sin^n x dx$. **(07 Marks)**

b. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of the family $r = a(1 - \cos\theta)$ **(07 Marks)**

8 a. Evaluate $\int_0^1 x^2 (1 - x^2)^{\frac{3}{2}} dx$. **(07 Marks)**

b. Solve : $\dfrac{dy}{dx} + \dfrac{1}{x} y = y^2 x$ **(06 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 min, the temperature of the air being 40°C. What will be the temperature of the body after 40 min from the original? **(07 Marks)**

Module – 5

9 a. Find the rank of matrix,

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

(06 Marks)

b. Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(07 Marks)**

c. Reduce $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ to canonical form by orthogonal transformation. **(07 Marks)**

10 a. Solve by Gauss elimination method:

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$x + y + z = 9$. **(06 Marks)**

b. Solve by LU decomposition method the equations,

$3x + 2y + 7z = 4$

$2x + 3y + z = 5$

$3x + 4y + z = 7$ **(07 Marks)**

c. Use power method to find the largest eigen value and the corresponding eigen vectors of,

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ taking initial eigen vectors $[1 \quad 1 \quad 1]^T$. Carryout 4 iterations. **(07 Marks)**

* * * * *