



CBCS SCHEME

18MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2021 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of $\cos t \cos 2t \cos 3t$. (06 Marks)
- b. If $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) - f(t)$, show that $L\{f(t)\} = \frac{1}{s^2} \tan h \left(\frac{as}{2} \right)$. (07 Marks)
- c. Find the Inverse Laplace transforms of :
- i) $\frac{2s+1}{s^2+6s+13}$ ii) $\frac{1}{3} \log \left(\frac{s^2+b^2}{s^2+a^2} \right)$. (07 Marks)

OR

- 2 a. Express the function $f(t)$ in terms of unit step function and find its Laplace transform, where
- $$f(t) = \begin{cases} 1, & 0 < t \leq 1 \\ t, & 1 < t \leq 2 \\ t^2, & t > 2 \end{cases}$$
- (06 Marks)
- b. Find the Inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$ using Convolution theorem. (07 Marks)
- c. Solve by the method of Laplace transforms, the equation $y'' + 4y' + 3y = e^{-t}$ given $y(0) = 0, y'(0) = 0$. (07 Marks)

Module-2

- 3 a. Obtain the Fourier series of the function $f(x) = x^2$ in $-\pi \leq x \leq \pi$. (06 Marks)
- b. Obtain the Fourier series expansion of
- $$f(x) = \begin{cases} x & , 0 < x < \pi \\ x - 2\pi & , \pi < x < 2\pi \end{cases}$$
- (07 Marks)
- c. Find the Cosine half range series for $f(x) = x(\ell - x), 0 \leq x \leq \ell$. (07 Marks)

OR

- 4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-\ell, \ell)$. (06 Marks)
- b. Find the sine half range series for
- $$f(x) = \begin{cases} x & , 0 < x < \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x < \pi \end{cases}$$
- (07 Marks)
- c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. (07 Marks)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Module-3

- 5 a. If $f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$. Find the Fourier transform of $f(x)$ and hence find value of $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$. (06 Marks)
- b. Find the Fourier Cosine transform of $f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4-x, & 1 < x < 4 \\ 0, & x > 4 \end{cases}$. (07 Marks)
- c. Find the Z - transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (07 Marks)

OR

- 6 a. Solve the Integral equation $\int_0^{\infty} f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1-\alpha, & 0 \leq \alpha \leq 1 \\ 0, & \alpha > 1 \end{cases}$ hence evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$. (06 Marks)
- b. Find the Inverse Z - transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (07 Marks)
- c. Using the Z - transform, solve $Y_{n+2} - 4Y_n = 0$, given $Y_0 = 0, Y_1 = 2$. (07 Marks)

Module-4

- 7 a. Using Taylor's series method, solve the Initial value problem $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$ at the point $x = 0.1$. Consider upto 4th degree term. (06 Marks)
- b. Use modified Euler's method to compute $y(0.1)$, given that $\frac{dy}{dx} = x^2 + y, y(0) = 1$ by taking $h = 0.05$. Consider two approximations in each step. (07 Marks)
- c. Given that $\frac{dy}{dx} = x - y^2$, find y at $x = 0.8$ with

x :	0	0.2	0.4	0.6
y :	0	0.02	0.0795	0.1762

By applying Milne's method. Apply corrector formula once. (07 Marks)

OR

- 8 a. Solve the following by Modified Euler's method $\frac{dy}{dx} = x + \sqrt{y}, y(0) = 1$ at $x = 0.4$ by taking $h = 0.2$. Consider two modifications in each step. (06 Marks)
- b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$. Compute $y(0.2)$ by taking $h = 0.2$ using Runge - Kutta method of order IV. (07 Marks)
- c. Given $\frac{dy}{dx} = (1+y)x^2$ and $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$, determine $y(1.4)$ by Adam's Bashforth method. Apply corrector formula once. (07 Marks)

Module-5

- 9 a. Given $y'' - xy' - y = 0$ with $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ using Runge – Kutta method. (06 Marks)
- b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (07 Marks)
- c. Prove that the geodesics on a plane are straight lines. (07 Marks)

OR

- 10 a. Find the curve on which functional $\int_0^1 [(y')^2 + 12xy] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized. (06 Marks)
- b. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data. Apply corrector formula once. (07 Marks)
- | | | | | |
|------|---|--------|--------|--------|
| x : | 1 | 1.1 | 1.2 | 1.3 |
| y : | 2 | 2.2156 | 2.4649 | 2.7514 |
| y' : | 2 | 2.3178 | 2.6725 | 3.0657 |
- c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is Catenary $y = c \cosh \left(\frac{x+a}{c} \right)$. (07 Marks)
