CBCS SCHEME

USN

17MAT41

Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Use Taylor's series method to find $y(1.5)$ from $y' = xy^{\frac{1}{3}}$, $y(1) = 1$, consider upto third order derivative term. **(06 Marks)**

b. Find $y(0.2)$ by using modified Euler's method given that $y' = x + \left|\sqrt{y}\right|$, $y(0) = 1$. Take $h = 0.2$ and carry out two modifications at each step. **(07 Marks)**

c. If $5xy' + y^2 = 2$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$ then find $y(4.4)$ by using Milne's method. **(07 Marks)**

2 a. Use Taylor's series method to find $y(1.02)$ from $y' = xy - 1$, $y(1) = 2$ consider upto fourth order derivative term. **(06 Marks)**

b. Use Runge-Kutta method to find $y(0.2)$ from $y' = \dfrac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ taking $h = 0.2$. **(07 Marks)**

c. Use Adam Bashforth method to find $y(0.4)$ from $y' = x + y^2$, $y(0) = 1$, $y(0.1) = 1.1$, $y(0.2) = 1.231$, $y(0.3) = 1.402$ **(07 Marks)**

3 a. Express $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(06 Marks)**

b. Find $y(0.1)$ by using Runge-Kutta method given that $y'' = x^3(y + y')$, $y(0) = 1$, $y'(0) = 0.5$ taking step length $h = 0.1$. **(07 Marks)**

c. If α and β are the roots of $J_n(\alpha) = 0$ then show that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

4 a. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\cos x$. **(06 Marks)**

b. Find $y(0.4)$ by using Milne's method given $y'' + y' = 2e^x$, $y(0) = 2$, $y'(0) = 0$, $y(0.1) = 2.01$, $y'(0.1) = 0.2$, $y(0.2) = 2.04$, $y'(0.2) = 0.4$, $y(0.3) = 2.09$, $y'(0.3) = 0.6$. **(07 Marks)**

c. State and prove Rodrigue's formula. **(07 Marks)**

5 a. Derive Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find the analytic function $f(z) = u + iv$ in terms of z given that $U = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. **(07 Marks)**

c. Evaluate $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ where C is the circle $|z| = 3$. **(07 Marks)**

6 a. If $f(z)$ is analytic function then prove that, $\left[\dfrac{\partial |f(z)|}{\partial x}\right]^2 + \left[\dfrac{\partial |f(z)|}{\partial y}\right]^2 = |f'(z)|^2$. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Find the bilinear transformation that maps the points $z = -1, i, 1$ onto the points $W = 1, i, -1$. Also find the invariant points. **(07 Marks)**

7 a. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation. Also find
(i) $P(x \le 1)$ (ii) $P(x > 1)$ (iii) $P(-1 < x \le 2)$

x	−3	−2	−1	0	1	2	3
P(x)	K	2K	3K	4K	3K	2K	K

(06 Marks)

b. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students where marks will be
(i) Less than 65 (ii) More than 75 (iii) Between 65 and 75 $(A(1) = 0.3413)$
(07 Marks)

c. The joint probability distribution for two random variables X and Y as follows:

X \ Y	−2	−1	4	6
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find : (i) E(X)E(Y) (ii) E(XY)
(iii) Covariance of (XY) (iv) Correlation of X and Y. (07 Marks)

8 a. Derive mean and variance of the exponential distribution. (06 Marks)

b. The joint probability distribution for two random variables X and Y as follows: (07 Marks)
Find (i) E(X) and E(Y)
(ii) E(XY)
(iii) Covariance (X, Y)
(iv) Correlation of X and Y.

X \ Y	−4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

c. In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to the defective. The blades are supplied in packets of 10. Using Poisson distribution find the approximate number of packets containing (i) No defective blade (ii) One defecting blade (iii) Two defective blades in a consignment of 10000 packets. (07 Marks)

9 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. (06 Marks)

b. A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure. $(t(11)_{00.05} = 2.2)$ (07 Marks)

c. Find the unique fixed probability for the regular stochastic matrix :

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

(07 Marks)

10 a. Define the terms : (i) Null hypothesis (ii) Type – I and Type II error.
(iii) Tests of significance. (06 Marks)

b. In experiments on pea breeding the following frequencies of seeds were obtained:

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory Predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment $\left(\psi^2_{0.05} = 7.815\right)$. (07 Marks)

c. A students study habits are as follows. If he studies one night, he is 30% sure to study the next night, on the other hand, if he does not study one night he is 60% sure not to study the next night as well. Find the transition matrix for the chain of his study. In the long run how often does he study?
(07 Marks)