CBCS SCHEME

USN

15MATDIP31

Third Semester B.E. Degree Examination, July/August 2021

Additional Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions.***

1 a. Express $\dfrac{(3+i)(1-3i)}{(2+i)}$ in the form x + iy. **(06 Marks)**

b. Find the modulus and amplitude of the complex number $1+\cos\alpha + i\sin\alpha$. **(05 Marks)**

c. If $\vec{a} = \hat{i}+2\hat{j}-2\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$, $\vec{c} = \hat{i}-2\hat{j}+2\hat{k}$, then find $\vec{a}\times(\vec{b}\times\vec{c})$. **(05 Marks)**

2 a. Prove that $\left[\dfrac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right]^n = \cos n\theta + i\sin n\theta$. **(06 Marks)**

b. Find the cube root of $1+i\sqrt{3}$. **(05 Marks)**

c. Show that the vectors $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$, $\vec{b} = -2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}+5\hat{k}$ are coplanar. **(05 Marks)**

3 a. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$. **(06 Marks)**

b. With usual notations prove that $\tan\phi = r\,.\, d\theta/dr$. **(05 Marks)**

c. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$ then show that $x\dfrac{\partial u}{\partial x} + y\,\partial u/\partial y = \sin 2u$. **(05 Marks)**

4 a. Find the n^{th} derivative of $\dfrac{x}{(x-2)(x-3)}$. **(06 Marks)**

b. Find the angle between the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(05 Marks)**

c. Given $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

5 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi/16} \cos^5(8x)\sin^6(16x)\, dx$. **(05 Marks)**

c. Evaluate $\int_1^2\int_1^3 x y^2\, dx\, dy$. **(05 Marks)**

6 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi} \dfrac{\sin^4\theta}{(1+\cos\theta)^2}\, d\theta$. **(05 Marks)**

c. Evaluate $\int_{-3}^{3}\int_0^1\int_1^2 (x+y+z)\, dx\, dy\, dz$. **(05 Marks)**

7 a. Find velocity and acceleration of a particle moving along the curve $\vec{r} = e^{-2t}\,\hat{i} + 2\cos 5t\,\hat{j} + 5\sin t\,\hat{k}$ at anytime t. Find their magnitudes at t = 0. (06 Marks)

b. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at (1, -1, 2). (05 Marks)

c. Show that $\vec{F} = (x + 3y)\,\hat{i} + (y - 3z)\,\hat{j} + (x - 2z)\,\hat{k}$ is Solenoidal. (05 Marks)

8 a. Find the unit tangent vector of the space curve $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$. (06 Marks)

b. If $\vec{F} = x^2y\,\hat{i} + yz^2\,\hat{j} + zx^2\,\hat{k}$, then find div (curl $\vec{F}$). (05 Marks)

c. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)\,\hat{i} + (x + cy + 2z)\,\hat{j} + (bx + 2y - z)\,\hat{k}$ is irrotational. (05 Marks)

9 a. Solve $\dfrac{dy}{dx} = 1 + \dfrac{y}{x} + \left(\dfrac{y}{x}\right)^2$. (06 Marks)

b. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. (05 Marks)

c. Solve $\dfrac{dy}{dx} = \dfrac{x^2 - 2xy}{x^2 - \sin y}$. (05 Marks)

10 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. (06 Marks)

b. Solve $(1 + xy)y\,dx + (1 - xy)\,x\,dy = 0$. (05 Marks)

c. Solve $x\dfrac{dy}{dx} + y = x^3y^6$. (05 Marks)

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