CBCS SCHEME

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18EC44

Fourth Semester B.E. Degree Examination, Feb./Mar.2022
Engineering Statistics & Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note:. Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define the following:
(i) Probability density function
(ii) Gaussian distribution. **(04 Marks)**

b. The probability density function of a random variable X is given by $f(x) = xe^{-x}$ for $X \geq 0$. Determine (i) CDF (ii) Evaluate $P(X \leq 1)$ (iii) $P[1 < X \leq 2]$ (iv) $P[X > 2]$ **(08 Marks)**

c. A random variable 'X' has a Poisson distribution with a mean of 3. Find $P[1 \leq X \leq 3]$. **(08 Marks)**

OR

2 a. Find the mean and variance of a random variable 'X' having a uniform distribution in the interval [a, b]. **(08 Marks)**

b. The normalized Gaussian random variable is given by $f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad -\infty < x < \infty$. Obtain the characteristic function for this random variable. **(08 Marks)**

c. Define the following:
(i) Laplace distribution function.
(ii) Binomial distribution function. **(04 Marks)**

Module-2

3 a. Define the following:
(i) Marginal densities.
(ii) Two variable expectations. **(04 Marks)**

b. Let 'X' and 'Y' be exponentially distributed random variable $f_X(x) = \begin{cases} \lambda e^{-\lambda X} & X \geq 0 \\ 0 & X < 0 \end{cases}$. **(08 Marks)**

c. Consider the two dimensional random variables X and Y, related to two dimensional random variables P and Q by $P = 4X + 2Y$, $Q = X + 2Y$, X and Y have zero means, and $\sigma_X^2 = 9$, $\sigma_Y^2 = 4$, $\rho_{XY} = -0.5$. Obtain ρ_{PQ}. **(08 Marks)**

OR

4 a. For sum of IID random variables prove that $\mu_w = n\mu_x$, $\sigma_W^2 = n\sigma_X^2$ and $\phi_W(jw) = \phi_X^n(jw)$. **(06 Marks)**

b. Define the following :
(i) Students 't' random variable.
(ii) Chi-square random variable. **(08 Marks)**

c. For averaging of random variables, for large "n' prove that $\mu_Y = \mu_X$ and $\sigma_Y^2 = \frac{\sigma_x^2}{h}$. **(06 Marks)**

Module-3

5 a. Define the following:
   (i) Random processes
   (ii) Stationary processes. **(04 Marks)**

b. Write the properties of Autocorrelation function. **(06 Marks)**

c. Show that the random process $X(t) = A\cos(\omega_C t + \theta)$ is wide sense stationary. 'θ' is uniformly distributed in the range $-\pi$ to π. **(10 Marks)**

OR

6 a. For the random process $X(t) = A\cos(\omega_C t + \theta)$, A and ω_C are constants. θ is a random variable, uniformly distributed between $\pm\pi$. Show that this process is ergodic. **(08 Marks)**

b. Determine the power spectral density of the random process $X(t) = A\cos(\omega_C t + \theta)$ and plot the same. Here θ is random variable uniformly distributed over 0 to 2π. Hence obtain average power of X(t). If the frequency becomes zero, X(t) = A i.e. a d.c. signal, then obtain power spectral density and autocorrelation function. **(08 Marks)**

c. A wide sense stationary random process X(t) is applied to a LTI system with impulse response $h(t) = ae^{-at}u(t)$. Find the mean value of the output Y(t) of the system if $E[X(t)] = 6$ and 'a' = 2. **(04 Marks)**

Module-4

7 a. Find the general solution of the linear system whose augmented matrix is as given below:

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}.$$ **(08 Marks)**

b. For what value of h will y be in the subspace of $\mathbb{R}^3$ spanned by V_1, V_2, V_3 if

$$V_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, V_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, V_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \text{and } Y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}.$$ **(04 Marks)**

c. $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and $u = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$. Determine if 'u' belongs to the null space of 'A'. **(04 Marks)**

d. Let $V_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $V_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$, $V_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, $V_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$, find a basis for the subspace 'W' spanned by $\{V_1, V_2, V_3, V_4\}$. **(04 Marks)**

OR

8 a. Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{bmatrix}$. **(06 Marks)**

b. The distance from a point 'Y' in R^n to a subspace 'W' is defined as the distance from 'Y' to the nearest point in W. Find the distance from Y to W = span $\{u_1, u_2\}$. Where $Y = \begin{bmatrix} -1 \\ -5 \\ 10 \end{bmatrix}$, $u_1 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. **(08 Marks)**

c. Let $W = \text{span}\{X_1\ X_2\}$, where $X_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis $\{V_1\ \&\ V_2\}$ for W. **(06 Marks)**

<u>Module-5</u>

9 a. Mention the properties of determinants. **(08 Marks)**

b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, show that matrix A is positive definite matrix. **(06 Marks)**

c. Find the eigen values of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. **(06 Marks)**

OR

10 a. Diagonalize the following matrix, if possible $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. **(10 Marks)**

b. Find a singular value of decomposition of, $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. **(10 Marks)**

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