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Hamiltonian Laceability in Some Classes of the Star Graphs

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Abstract*— The graph G is Hamiltonian laceable [2] if there exists a Hamiltonian path between every pair of distinct vertices in it at an odd distance. G is Hamiltonian-t-laceable (t*-laceable) if there exists a Hamiltonian path in G between every pair (at least one pair) of vertices u and v in G with the property $d(u,v)=t$ In this paper, we discuss the Hamiltonian laceability properties of the graph $G*v$, where G is the Star graph $G=K_{1,n}$, $(n \geq 3)$. We also explore the Hamiltonian Laceability properties of the subdivision graph G^{+}.

***Index Terms*—Hamiltonian path, Hamiltonian laceability, Hamiltonian-t-laceable path, i-Hamiltonian laceability.**

I. INTRODUCTION

Let G be a finite, simple connected undirected graph. Let *u* and *v* be two vertices in G. The order of G denoted by $O(G)$ is the cardinality of the vertices of G. The distance between *u* and *v* denoted by $d(u,v)$ is the length of a shortest *u*-*v* path in G. G is *Hamiltonian Laceable* if there exists a Hamiltonian path between every pair of distinct vertices in it at an odd distance. G is *Hamiltonian-t-laceable* if there exists a Hamiltonian path between every pair of vertices *u* and *v* in G with the property $d(u,v)=t$ and *Hamiltonian-t*-laceable* [2] if there exists a Hamiltonian path between at least one such pair with the property $d(u,v)=t$, where *t* is a positive integer such that $1 \leq t \leq diamG$. Hamiltonian laceability in the brick product of cycles was explored by B. Alspach, C.C. Chen and Kevin McAvaney in [1] where the authors proved the laceability in the brick product of odd cycles. Hamiltonian –t-laceability in the brick product of even cycles was studied by Leena. N. Shenoy and R. Murali in [2]. In [3], Girisha. A and R. Murali have studied Hamiltonian-*t**-laceability of 4-regular graphs. In this paper we study the Hamiltonian-*t**-laceability properties of the graph extended star graph $G*v$ and the subdivision graph G^{+}.

Definition1.1: Let $G=K_{1,n}$ be the star graph and $v \in V(K_{1,n})$. The graph $G*v$ is obtained from G by replacing the vertex *v* by a cycle of length *n* and joining the vertices of the cycle to the former neighbors of *v* as shown in Fig.1.

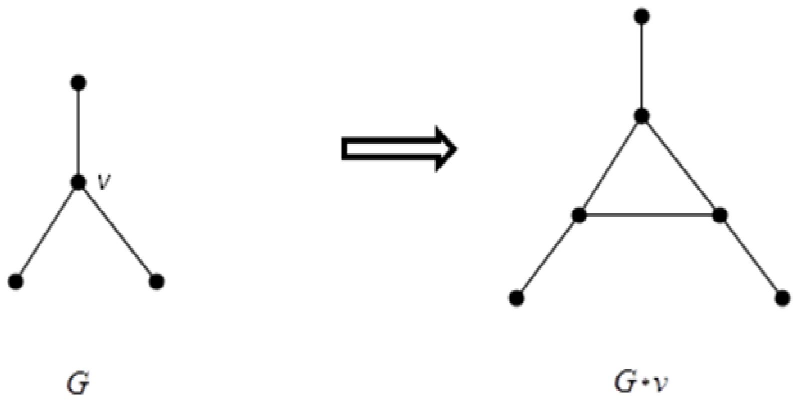

Fig. 1: The Graphs G and $G*v$

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Definition1.2: Let G be a complete graph. The subdivision graph obtained by inserting a vertex of degree two into any one edge of G and we denote it by G^+.

When the inserted vertex in a subdivision of G is specified, say *u*, we denote by $G(u)$ a graph with $V(G(u)) = V(G) \cup \{u\}$ and $E(G(u)) = (E(G) - xy) \cup \{xu, xy\}$ where $xy \in E(G)$.

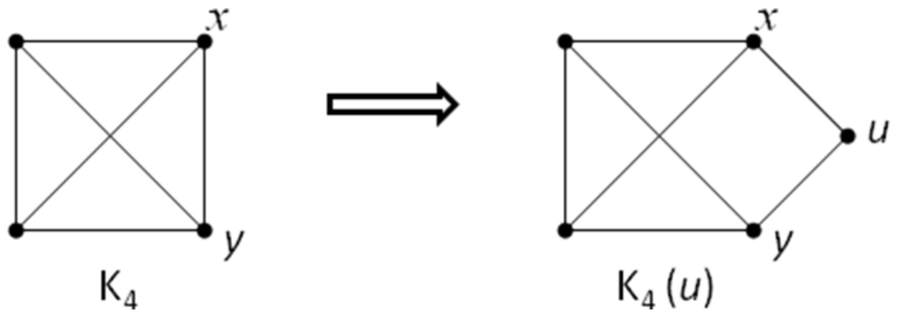

Fig. 2: The Graph K$_4$(*u*)

Definition1.3: Let G be a connected graph of order n and let $h_p(G)$ be the length of a Hamiltonian path [4] between any two distinct vertices in G. A Hamiltonian path in G is called a 0-*Hamiltonian path* if $h_p(G) = n - 1$ and a *1-Hamiltonian path* if $h_p(G) = n$

Definition1.4: Let *i* be a non-negative integer. A connected graph G of order n is called *i-Hamiltonian-t-laceable* if there exists in G, a *i-Hamiltonian path* between every pair of distinct vertices *u* and *v* with the property $d(u,v) = t$, $1 \le t \le diamG$.

Definition1.5: A connected graph G of order n is called *i-Hamiltonian-t*-laceable* if there exists in G, a *i-Hamiltonian path* [4] between at least one pair of distinct vertices *u* and *v* with the property $d(u,v) = t$, $1 \le t \le diamG$.

Definition 1.6: Let $G = K_{1,n}$, $n \ge 3$, be the star graph of order n. Then the extended star graph $K_{1,n,n}$ is obtained by inserting a star graph of order $n-1$ to each pendent vertex of $K_{1,n}$.

II. RESULTS

Theorem 2.1: The graph $G = K_{1,n} * v$, $n \ge 3$ is i-Hamiltonian-1*-laceable for $i = n$.

Proof: Let us denote the vertices of $K_{1,n} * v$ by $a_1, a_2, a_3, a_4, a_5 \ldots\ldots\ldots a_n$ and $b_1, b_2, b_3, b_4, b_5 \ldots\ldots\ldots b_n$. Here we need to establish the following case to show that G is i-Hamiltonian-1*-laceable.

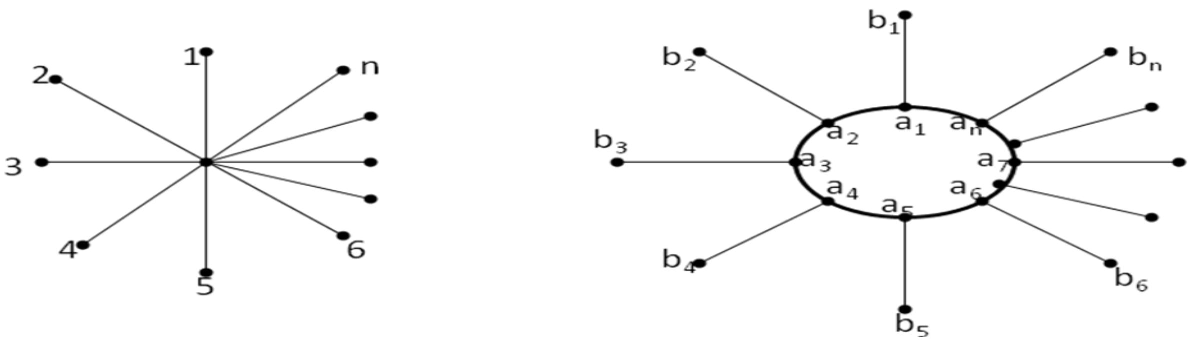

Fig. 2: The Graphs $k_{1,n}$ and $k_{1,n} * v$

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In G, $d(b_1, a_1) = 1$ and the path

$P:(b_1,a_2)\cup(a_2,b_2)\cup(b_2,a_3)\cup(a_3,b_3)\cup(b_3,a_4)\cup(a_4,b_4)\cup(b_4,a_5)\cup(a_5,b_5)\cup(b_5,a_6)\cup(a_6,b_7)..........(a_{n-1},b_{n-1})\cup(b_{n-1},a_n)\cup(a_n,b_n)\cup(b_n,a_1)$ *is a Hamiltonian path from* b_1 *to* a_1 in G.

Hence the proof ■

Theorem 2.2: The $G = k_{1,n} * v$ ***, n ≥ 3 is i-Hamiltonian-2*-laceable for i=n-1.***

Proof: Let us denote the vertices of $K_{1,n} * v$ by $a_1, a_2, a_3, a_4, a_5..........a_n$ and $b_1, b_2, b_3, b_4, b_5..........b_n$. Here we need to establish the following case to show that G is i-Hamiltonian-2*-laceable. In G, $d(b_1, a_2) = 2$ and the path

$P:(a_2,b_2)\cup(b_2,a_3)\cup(a_3,b_3)\cup(b_3,a_4)\cup(a_4,b_4)\cup(b_4,a_5)\cup(a_5,b_5)\cup(b_5,a_6)\cup(a_6,b_6)\cup(b_6,a_7)\cup(a_7,b_7)\cup..............\cup(a_{n-1},b_{n-1})(b_{n-1},a_n)(a_n,b_n)(b_n,a_1)(a_1,b_1)$ is a Hamiltonian path from b_1 *to* a_2 *in* G.

Hence the proof ■

Theorem 2.3: Let G ***be the complete graph of order*** n ***(***$n \geq 3$***). Then*** G^+ ***is 1-Hamiltonian-2*-laceable.***

Proof: Let $G = k_n$ $(n \geq 3)$ be the complete graph and G^+ be the subdivision graph obtained by inserting a vertex u of degree two into any edge of G with the end vertices x and y such that $d(x, y) = 2$. G^+ has $n+1$ vertices and ${}^nC_2 + 1$ edges.

Let $u, x, a_1, a_2, a_3, a_4, a_5.....................a_{n-2}, y$ be the vertices of G^+.

Then the path

$P:(x,a_1)\cup(a_1,a_2)\cup(a_2,a_3)\cup(a_3,a_4)\cup(a_4,a_5)\cup(a_5,a_6)\cup...................\cup(a_{n-3},a_{n-2})\cup(a_{n-2},u)\cup(u,y)$ is a Hamiltonian - 2 *-laceable path from x to y.

Hence the proof ■

Theorem 2.4: The graph $k_{1,n,n}$ ***is i-Hamiltonian-1*-laceable for*** $i = O(k_{1,n,n}) - 3$.

Proof: Let $G = k_{1,n}$ be a star graph of order n and $G_1 = k_{1,n,n}$ be a extended star graph with vertices $b_1, b_2, b_3, b_4, b_5..........b_{n(n-3)} - b_{n(n-2)} - b_{n(n-1)}$ and $a_1, a_2, a_3, a_4, a_5..........a_n$ and a parent vertex v.

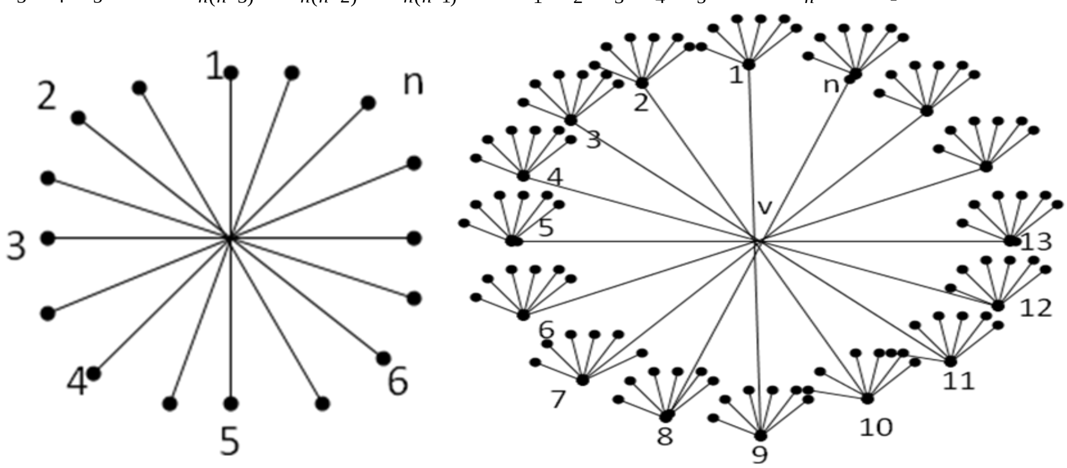

Fig. 3: The graph $k_{1,n}$ **and** $k_{1,n,n}$

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In $G_1, \ d(a_1, v) = 1$ and the path

$$P:(a_1,b_1)\cup(b_1,b_2)\cup(b_2,b_3)\cup.........\cup(b_{n-1},a_2)\cup(a_2,b_n)\cup(b_n,b_{n+1})\cup(b_{n+1},b_{n+2})\cup(b_{n+2},b_{n+3})\cup$$
$$(b_{n+3},b_{n+4})\cup.............\cup(b_{(2n-3)},b_{(2n-2)})\cup(b_{(2n-2)},a_3)\cup(a_3,b_{(2n-1)})\cup(b_{(2n-1)},b_{2n})\cup(b_{2n},b_{2n+1})\cup$$
$$............\cup(b_{(3n-4)},b_{(2n-3)})\cup(b_{(2n-3)},a_4)\cup(a_4,b_{(2n-2)})\cup(b_{(2n-2)},b_{(2n-1)})\cup(b_{(2n-1)},b_{2n})\cup.................$$
$$\cup(b_{(3n+2)},b_{(3n+3)})\cup(b_{(3n+3)},a_5)\cup(a_5,b_{(3n+4)})\cup(b_{(3n+4)},b_{(3n+5)})\cup.............\cup(b_{(4n+1)},b_{(4n+2)})\cup...........$$
$$........\cup(a_n,b_{[(n-2)n+2]})\cup(b_{[(n-2)n+2]},b_{[(n-2)n+3]})\cup(b_{[(n-2)n+3]},b_{[(n-2)n+3]})\cup(b_{[(n-2)n+3]},b_{[(n-2)n+4]})\cup.............$$
$$\cup(b_{[(n-2)n+n]},v)\ is\ a\ i-hamiltonian-1^{*}-laceable\ \ path\ \ from\ a_1\ to\ v\ with\ i=O(k_{1,n,n})-3.$$

Hence the proof

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