

# CBCS SCHEME

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17MAT11

Learning Resource Centre  
Acharya Institute & Technology

First Semester B.E. Degree Examination, Feb./Mar. 2022

## Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- Find the  $n^{\text{th}}$  derivative of  $y = \cos 2x \cos 3x$ . (06 Marks)
  - Find the angle of intersection between the curves  $r = a \operatorname{cosec}^2\left(\frac{\theta}{2}\right)$  and  $r = b \sec^2\left(\frac{\theta}{2}\right)$ . (07 Marks)
  - Find the radius of curvature of the curve  $x^4 + y^4 = 2$  at the point (1, 1). (07 Marks)

OR

- If  $y = \tan^{-1}x$ , prove that  $(1 + x^2) y_{n+2} + 2(n+1) x y_{n+1} + n(n+1) y_n = 0$ . (06 Marks)
  - Derive  $\tan \phi = r \frac{d\theta}{dr}$  with usual notations. (07 Marks)
  - Prove that the radius of curvature of the curve  $r^n = a^n \cos n\theta$ . (07 Marks)

### Module-2

- Expand  $\tan^{-1}x$  upto and including  $x^5$  using Maclaurin's series. (06 Marks)
  - If  $u = \log_e\left(\frac{x^3 + y^3}{x^2 + y^2}\right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$ . (07 Marks)
  - If  $u = \frac{x_2 x_3}{x_1}$ ,  $v = \frac{x_1 x_3}{x_2}$ ,  $w = \frac{x_1 x_2}{x_3}$ , prove that  $J\left(\frac{u, v, w}{x_1, x_2, x_3}\right) = 4$ . (07 Marks)

OR

- Evaluate  $\lim_{x \rightarrow 0} \left( \frac{x e^x - \log(1+x)}{x^2} \right)$ . (06 Marks)
  - Expand  $f(x) = \log_e x$  about  $x = 1$  upto the term containing third degree terms using Taylor's series. (07 Marks)
  - If  $u = f(r, s, t)$  and  $r = \frac{x}{y}$ ,  $s = \frac{y}{z}$ ,  $t = \frac{z}{x}$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ . (07 Marks)

### Module-3

- A particle moves along the curve  $x = t^3 + 1$ ,  $y = t^2$ ,  $z = t + 5$ ,  $t$  – time, find the components of the velocity and acceleration at  $t = 2$  in the direction of  $\hat{i} + 3\hat{j} + 2\hat{k}$ . (06 Marks)
  - Find  $\operatorname{div} F$  and  $\operatorname{curl} F$  if  $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ . (07 Marks)
  - Show that  $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$  is irrotational field. Find  $\phi$  such that  $F = \nabla\phi$ . (07 Marks)

OR

- 6 a. Find the value of  $a$  for which  $f = (x + 3y)i + (y - 2z)j + (x + az)k$  is solenoidal. (06 Marks)  
 b. Prove that  $\text{div}(\text{curl } A) = 0$ . (07 Marks)  
 c. If  $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ , find the value of  $\text{curl}(\text{curl } A)$ . (07 Marks)

Module-4

- 7 a. Obtain the reduction formula of  $\int \cos^n x \, dx$  and hence evaluate  $\int_0^{\pi/2} \cos^n x \, dx$ . (06 Marks)  
 b. Solve  $(xy + y^2) dx + (x + 2y - 1) dy = 0$ . (07 Marks)  
 c. Find the orthogonal trajectories of the curve  $r = 4a \sec\theta \tan\theta$ ,  $a$  is the parameter. (07 Marks)

OR

- 8 a. Evaluate  $\int_0^1 x^{3/2} (1-x)^{3/2} dx$  (06 Marks)  
 b. Solve  $(1 + xy^2)xy \frac{dy}{dx} = 1$  (07 Marks)  
 c. A body originally at  $80^\circ\text{C}$  cools down to  $60^\circ\text{C}$  in 20min. The temperature of the air being  $40^\circ\text{C}$ . What will be the temperature of the body after 40min from the original? (07 Marks)

Module-5

- 9 a. Solve by Gauss Elimination method the system of equations  
 $x + 2y = 3 - z$   
 $2x + 3y + 3z = 10$   
 $3x - y + 2z = 13$  (06 Marks)  
 b. Find the largest Eigen value and the corresponding Eigen vector of the matrix  
 $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$  by power method choosing  $[1 \ 0 \ 0]^T$  as initial vector for obtaining  
 4 approximations. (07 Marks)  
 c. Reduce quadratic form  $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2yz$  to canonical form, using orthogonal transformation. (07 Marks)

OR

- 10 a. Find the rank of the matrix  $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$  (06 Marks)  
 b. Reduce the matrix  $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$  into diagonal form. (07 Marks)  
 c. Find the inverse transformation of  
 $u_1 = 9v_1 + 6v_2$   
 $u_2 = 10v_1 - 2v_2$ . (07 Marks)

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