CBCS SCHEME

USN

18MAT21

Second Semester B.E. Degree Examination, Feb./Mar.2022
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2, -1, 2)$. **(06 Marks)**

b. Find the divergence and curl of the vector $\vec{F}$ if $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$. **(07 Marks)**

c. Show that $\vec{F}=(y+z)i+(z+x)j+(x+y)k$ is irrotational and also find a scalar function ϕ such that $\vec{F}=\nabla\phi$. **(07 Marks)**

OR

2 a. Verify Green's theorem for $\int_C (xy+y^2)dx+x^2dy$, where C is the bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Using Stoke's theorem, evaluate $\int_C xydx+xy^2dy$, where C is the square in the x-y plane with vertices (1, 0)(-1, 0)(0, 1)(0, -1). **(07 Marks)**

c. Using Gauss divergence theorem, evaluate $\iint_C \vec{F}\,\overline{n}\,ds$ over the entire surface of the region above xy-plane bounded by the cone $z^2=x^2+y^2$ and the plane $z = 4$, where $\vec{F}=4xz\,\vec{i}+xyz^2\,\vec{j}+3z\,\vec{k}$. **(07 Marks)**

Module-2

3 a. Solve $(D^2-4D+13)y=\cos 2x$, where $D=\frac{d}{dx}$. **(06 Marks)**

b. Solve $(D^2-2D+1)y=\frac{e^x}{x}$, by the method of variation of parameter, where $D=\frac{d}{dx}$. **(07 Marks)**

c. Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. **(07 Marks)**

OR

4 a. Solve $(D-2)^2y=8(e^{2x}+\sin 2x)$, where $D=\frac{d}{dx}$. **(06 Marks)**

b. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin 2[\log(1+x)]$. **(07 Marks)**

c. The differential equation of the displacement x(t) of a spring fixed at the upper end and a weight at its lower end is given by $10\frac{d^2x}{dt^2}+\frac{dx}{dt}+200x=0$. The weight is pulled down 0.25 cm, below the equilibrium position and then released. Find the expression for the displacement of the weight from its equilibrium position at any time t during its first upward motion. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary constants form, $(x-a)^2+(y-b)^2+z^2=C^2$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y}=\sin x \sin y$ for which $\frac{\partial z}{\partial y}=-2\sin y$ when x = 0 and z = 0 if y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one-dimensional heat equation in the standard form. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function from $z=f(x+ct)+g(x-ct)$ **(06 Marks)**

b. Solve $(y-z)p+(z-x)q=(x-y)$. **(07 Marks)**

c. Solve one dimensional wave equation, using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Test for the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{n!}{(n^n)^2}$. **(06 Marks)**

b. Solve Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $x^4-2x^3+3x^2-4x+5$ interms of legendre polynomial. **(07 Marks)**

OR

8 a. Discuss the nature of the series, $\frac{1}{2}+\left(\frac{2}{3}\right)x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+........$ **(06 Marks)**

b. With usual notation, show that

(i) $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$

(ii) $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ **(07 Marks)**

c. Use Rodrigues formula to show that $P_4(\cos\theta)=\frac{1}{64}[35\cos 4\theta+20\cos 2\theta+9]$. **(07 Marks)**

Module-5

9 a. Find a real root of the equation $\cos x-3x+1=0$, correct to 3 decimal places using regula falsi method. **(06 Marks)**

b. Use an appropriate interpolation formula to compute f(42) using the following data:

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

(07 Marks)

c. Evaluate $\int_{4}^{5.2} \log x dx$ by using Weddle's rule, divided into six equal parts. **(07 Marks)**

OR

10 a. Find a real root of the equation, $x\sin x + \cos x = 0$ near $x = \pi$, correct to four decimal places. Using Newton-Raphson method. **(06 Marks)**

b. Find f(9) from the day by Newton's divided difference formula. **(07 Marks)**

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

c. By using Simpson's $\frac{1}{3}^{rd}$ rule $\int_{0}^{1} \frac{dx}{1+x^2}$ dividing interval (0, 1) into six equal parts and hence find approximate value of π. **(07 Marks)**

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