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15MATDIP31

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Express $\frac{(3+i)(1-3i)}{2+i}$ in the form $x + iy$. (06 Marks)
- b. Find the modulus and amplitude of $1 - \cos\alpha + i\sin\alpha$. (05 Marks)
- c. Solve $z^3 + 1 = 0$ (05 Marks)

OR

- 2 a. Prove that $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$. (06 Marks)
- b. Show that if $\vec{a} = i + j + 2k$, $\vec{b} = 2i - j + k$ then $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$. (05 Marks)
- c. If \vec{a} , \vec{b} , \vec{c} are any three non-zero vectors, then prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. (05 Marks)

Module-2

- 3 a. Find the n^{th} derivative of $e^{ax} \sin(bx + c)$. (06 Marks)
- b. If $y = e^{m \sin^{-1} x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (05 Marks)
- c. Show that the following pair of curves intersect orthogonally $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$. (05 Marks)

OR

- 4 a. Find the pedal equation to the curve $r = a(1 + \sin\theta)$. (06 Marks)
- b. If $u \cdot (x + y) = x^2 + y^2$, then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$. (05 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (05 Marks)

Module-3

- 5 a. Obtain the reduction formula for $\int \cos^n x dx$, where n being a +ve integer. (06 Marks)
- b. Evaluate $\int_0^1 x^{3/2} (1-x)^{3/2} dx$ (05 Marks)
- c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ (05 Marks)

OR

- 6 a. Evaluate $\int_0^{\pi} x \sin^6 x dx$ (06 Marks)
- b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}} dx$ (05 Marks)
- c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$ (05 Marks)

Module-4

- 7 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4$, $z = 3t - 5$, find the components of velocity and acceleration in the direction of the vector $i - 3j + 2k$ at $t = 2$. (06 Marks)
- b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, -2)$. (05 Marks)
- c. Find the constants 'a' and 'b' such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. (05 Marks)

OR

- 8 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, +1)$ in the direction $2i - j - 2k$. (06 Marks)
- b. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then prove that $\nabla(r^n) = nr^{n-2}\vec{r}$. (05 Marks)
- c. Show that the vector $\vec{F} = (-2x^2y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2y^2)k$ is solenoidal. (05 Marks)

Module-5

- 9 a. Solve $(x^2 - y^2)dx - xydy = 0$. (06 Marks)
- b. Solve $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$. (05 Marks)
- c. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (05 Marks)

OR

- 10 a. Solve $(x + 2y - 3)dx - (2x + y - 3)dy = 0$. (06 Marks)
- b. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$. (05 Marks)
- c. Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (05 Marks)
