CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find a unit vector normal to the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$. Also find the sine of the angle between them. **(08 Marks)**

b. Express $\frac{1+2i}{1-3i}$ in the form of a + ib. **(06 Marks)**

c. Express $\sqrt{3} + i$ in the polar form and hence find its modulus and amplitude. **(06 Marks)**

OR

2 a. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$. **(08 Marks)**

b. If $\vec{a} = 3\hat{i} - 7\hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} - 5\hat{j} + 10\hat{k}$. Find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. **(06 Marks)**

c. Prove that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are co-planar. **(06 Marks)**

Module-2

3 a. If $y = e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. **(08 Marks)**

b. Find the angle between the curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$. **(06 Marks)**

c. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(06 Marks)**

OR

4 a. Using Maclaurin's series expand $\sin x$ upto the term containing x^5. **(08 Marks)**

b. Find the pedal equation of the curve $r^m \cos m\theta = a^m$. **(06 Marks)**

c. If $u = x + y + z$, $v = y + z$, $w = z$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x\, dx (n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^1 \frac{x^6}{\sqrt{1-x^2}} dx$ by taking $x = \sin\theta$. **(06 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\, dxdydz$. **(06 Marks)**

OR

6 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x \, dx \; (n > 0)$ **(08 Marks)**

b. Evaluate $\int_0^{\pi/2} \cos^4 \theta \, d\theta$ using reduction formula. **(06 Marks)**

c. Evaluate $\int_0^1 \int_0^2 xy \, dy \, dx$. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$. **(08 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, –2, 1) in the direction of the vector $2\hat{i} - \hat{j} + 2\hat{k}$. **(06 Marks)**

c. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is solenoidal. **(06 Marks)**

OR

8 a. Find $\operatorname{div}\vec{F}$ and $\operatorname{crul}\vec{F}$, where $\vec{F} = (3x^2 - 3yz)\hat{i} + (3y^2 - 3xz)\hat{j} + (3z^2 - xy)\hat{k}$. **(08 Marks)**

b. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$, find grad $(\operatorname{div}\vec{F})$ at (2, –1, 0). **(06 Marks)**

c. Find the constants a, b, c such that the vector, $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve : $(x^2 - y^2)dx - xy\,dy = 0$. **(08 Marks)**

b. Solve : $(1 + y^2)dx = (\tan^{-1} y - x)dy$. **(06 Marks)**

c. Solve : $(x^2 + y^2 + 1)dx + 2xy\,dy = 0$. **(06 Marks)**

OR

10 a. Solve : $x^2y\,dx - (x^3 + y^3)dy = 0$. **(08 Marks)**

b. Solve : $\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\}dx + (x + \log x - x\sin y)dy = 0$. **(06 Marks)**

c. Solve : $(x+1)\dfrac{dy}{dx} - ye^{3x}(x+1)^2\dfrac{dy}{dx} + \dfrac{y}{x} = 1$. **(06 Marks)**

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