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18EC44

Fourth Semester B.E. Degree Examination, July/August 2022
Engineering Statistics and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define an uniform random variable. Obtain the characteristic function of an uniform random variable and using the characteristic function derive its mean and variance. (08 Marks)
- b. If the probability density function of a random variable is given by

$$f_X(x) = \begin{cases} C \exp(-x/4), & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value that C must have and evaluate $F_X(0.5)$. (06 Marks)

- c. The density function of a random variable is given as

$$f_X(x) = a e^{-bx} \quad x \geq 0$$

Find the characteristic function and the first two moments. (06 Marks)

OR

- 2 a. Define a Poisson random variable. Obtain the characteristic function of a Poisson random variable and hence find mean and variance using the characteristic function. (08 Marks)
- b. Suppose 'X' is a general discrete random variable with following probability distribution. Calculate mean and variance for X.

X	0	1	3	5	7
P(X)	0.05	0.2	0.6	0.1	0.05

(06 Marks)

- c. The number of defects in a thin copper wire follows Poisson distribution with mean of 2.3 defects per millimeter. Determine the probability of exactly two defects per millimeter of wire. (06 Marks)

Module-2

- 3 a. Define and explain Central Limit theorem and show that the sum of the two independent Gaussian random variables is also Gaussian. (08 Marks)
- b. Let 'X' and 'Y' be exponentially distributed random variable with

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Then obtain the characteristic function and Pdf of $W = X + Y$. (06 Marks)

- c. Determine a constant b such that the given function is a valid joint density function.

$$f_{XY}(x, y) = \begin{cases} b(x^2 + 4y^2) & 0 \leq |x| < 1 \text{ and } 0 \leq y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(06 Marks)

OR

- 4 a. Explain briefly the following random variables :
(i) Chi-square Random Variable
(ii) Rayleigh Random Variable. (04 Marks)

- b. The joint density function of two random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} \frac{(x+y)^2}{40}, & -1 < x < 1 \text{ and } -3 < y < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) the variances of X and Y (ii) the correlation coefficient. (08 Marks)

- c. Gaussian random variables X_1 and X_2 whose $\bar{X}_1 = 2$, $\sigma_{X_1}^2 = 9$, $\bar{X}_2 = -1$, $\sigma_{X_2}^2 = 4$ and

$C_{X_1X_2} = -3$ are transformed to new random variables Y_1 and Y_2 such that

$$\begin{aligned} Y_1 &= -X_1 + X_2 \\ Y_2 &= -2X_1 - 3X_2 \end{aligned}$$

Find (i) \bar{X}_1^2 (ii) \bar{X}_2^2 (iii) $\rho_{X_1X_2}$ (iv) $\sigma_{Y_1}^2$ (v) $\sigma_{Y_2}^2$ (vi) $C_{Y_1Y_2}$ (vii) $\rho_{Y_1Y_2}$ (08 Marks)

Module-3

- 5 a. With the help of an example, define Random process and discuss distribution and density functions of a random process. Mention the differences between Random variable and Random process. (08 Marks)
- b. Define the Autocorrelation function of the random process $X(t)$ and discuss its properties. (06 Marks)
- c. A stationary ergodic random process has the autocorrelation function with periodic components as $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$
Find the mean and variance of $X(t)$. (06 Marks)

OR

- 6 a. The autocorrelation function of a wide sense stationary process.

$$R_X(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & -T \leq |\tau| \leq T \\ 0, & \text{elsewhere} \end{cases}$$

Obtain the Power Spectral Density of the process. (06 Marks)

- b. Show that the random process $X(t) = A \cos(\omega_c t + \theta)$ is wide sense stationary. Here θ is uniformly distributed in the range $-\pi$ to π . (08 Marks)

- c. $X(t)$ and $Y(t)$ are independent, jointly wide sense stationary random processes given by

$$X(t) = A \cos(\omega_1 t + \theta_1)$$

$$Y(t) = B \cos(\omega_2 t + \theta_2)$$

If $W(t) = X(t) \cdot Y(t)$ then find the Autocorrelation function $R_W(\tau)$. (06 Marks)

Module-4

- 7 a. Define vector subspaces and explain the four fundamental subspaces. (06 Marks)
- b. Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$ form a basis of \mathbb{R}^3 . (06 Marks)
- c. Apply Gram-Schmidt process to the vectors $v_1 = (2, 2, 1)$, $v_2 = (1, 3, 1)$, $v_3 = (1, 2, 2)$ to obtain an orthonormal basis for $v_3(\mathbb{R})$ with the standard inner product. (08 Marks)

OR

- 8 a. Determine the null space of each of the following matrices:

(i) $A = \begin{bmatrix} 2 & 0 \\ -4 & 10 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -7 \\ -3 & 21 \end{bmatrix}$

(06 Marks)

- b. Determine whether the vectors $(2, -2, 4)$, $(3, -5, 4)$ and $(0, 1, 1)$ are linearly dependent or independent. (06 Marks)

- c. Find the QR-decomposition for the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 7 \\ 0 & -1 & -1 \end{bmatrix}$$

and write the result in the form of $A = QR$.

(08 Marks)

Module-5

9 a. If $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

find eigen values and corresponding eigen vectors for matrix A.

(08 Marks)

- b. Diagonalize the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

(08 Marks)

- c. What is the positive definite matrix? Mention the methods of testing positive definiteness. (04 Marks)

OR

- 10 a. Factorize the matrix A into $A = U \Sigma V^T$ using SVD.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

(08 Marks)

- b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ show that A is positive definite matrix.

(04 Marks)

- c. Find a matrix P, which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to diagonal form. (08 Marks)
