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21MAT11

First Semester B.E. Degree Examination, July/August 2022
Calculus and Differential Equations

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notation prove that $\tan \phi = r \left(\frac{d\theta}{dr} \right)$. (06 Marks)
- b. Find the angle between the curves $r = 2 \sin \theta$ and $r = 2 \cos \theta$. (07 Marks)
- c. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4} \right)$. (07 Marks)

OR

- 2 a. With usual notation prove that
$$\rho = \frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2}$$
 (06 Marks)
- b. Find the radius of curvature for the curve $r^n = a^n \sin n \theta$. (07 Marks)
- c. Show that $r = 4 \sec^2 \theta/2$ and $r = 9 \operatorname{cosec}^2 \theta/2$ the pair of curves cut orthogonally. (07 Marks)

Module-2

- 3 a. Expand $Y = \log(1 + \sin x)$ upto the term contains x^4 by Maclaurin's series expansion. (06 Marks)
- b. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, show that $6u_x + 4u_y + 3u_z = 0$. (07 Marks)
- c. Show that the function $f(x, y) = x^3 + y^3 - 63x - 63y + 12xy$ is maximum at $(-7, -7)$. (07 Marks)

OR

- 4 a. Evaluate $\operatorname{Lt}_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$. (06 Marks)
- b. If $u = x^3 + y^3$ where $x = a \cos t$, $y = b \sin t$ find $\frac{dU}{dt}$. (07 Marks)
- c. If $U = e^x \cos y$, $V = e^x \sin y$. Find $\frac{\partial(u, v)}{\partial(x, y)}$. (07 Marks)

Module-3

- 5 a. Solve for P : $xp^2 + (y - x)p - y = 0$; where $p = \frac{dy}{dx}$. (06 Marks)
- b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. (07 Marks)
- c. Solve $(x^2 + y^2 + x) dx + xy dy = 0$. (07 Marks)

OR

- 6 a. Solve ; $[1 + \log x + \log y] dx + \left(1 + \frac{x}{y}\right) dy = 0$. (06 Marks)
- b. Solve : $p = \sin(y - xp)$. Also find its singular solutions. (07 Marks)
- c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C . What will be the temperature of the body after 40 minutes from the original? (07 Marks)

Module-4

- 7 a. Solve : $\frac{d^4y}{dx^4} - 4\frac{d^2y}{dx^2} + 4y = 0$. (06 Marks)
- b. Solve : $(D^2 + 4)y = e^x + \cos 2x$. (07 Marks)
- c. Using Variation of parameter method, solve
 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$. (07 Marks)

OR

- 8 a. Solve : $(D^2 - 1)y = 1 + x + x^2$. (06 Marks)
- b. Solve : $(D^2 + D + 1)y = (1 - e^x)$. (07 Marks)
- c. Solve : $(1+x)^2 \frac{dy^2}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin[\log(1+x)]$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix :

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

(06 Marks)

- b. Find for what values of
- λ
- and
- μ
- the system of linear equations :

$$x + y + z = 6$$

$$x + 2y + 5z = 10$$

$$2x + 3y + \lambda z = \mu$$

has

- i) a unique solution
- ii) no solution
- iii) Infinitely many solutions.

(07 Marks)

- c. Solve the system of equations :

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22$$

by Gauss Seidel method taking $(0, 0, 0)$ as an initial approximate root (carry out 3 iteration).

(07 Marks)

OR

- 10 a. Find the rank of the matrix :

$$\begin{bmatrix} 221 & 22 & 23 & 24 \\ 22 & 23 & 24 & 25 \\ 23 & 24 & 25 & 26 \\ 24 & 25 & 26 & 27 \end{bmatrix}$$

(06 Marks)

- b. Solve the system of equations by Gauss – Jordan method,

$$x + y + z = 9$$

$$x - 2y + 3z = 8$$

$$2x + y - z = 3.$$

(07 Marks)

- c. Using Rayleigh power method find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

by taking $(1, 1, 1)^T$ as initial eigen vector (carryout 5 iterations).

(07 Marks)
