

Second Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. (05 Marks)
- b. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 2^x$. (05 Marks)
- c. Solve $y'' - 2y' + y = e^x \log x$ by the method of variation of parameters. (06 Marks)

OR

- 2 a. Solve $(D^2 - 2D + 4)y = e^x \cos x$. (05 Marks)
- b. Solve $(D^2 + 4)y = x^2 + \sin 2x$. (05 Marks)
- c. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$ by the method of undetermined coefficients. (06 Marks)

Module-2

- 3 a. Solve $x^2y'' + xy' + y = 2\cos^2(\log x)$. (05 Marks)
- b. Solve $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$. (05 Marks)
- c. By reducing into Clairaut's form, obtain the general and singular solution of $xp^3 - yp^2 + 1 = 0$. (06 Marks)

OR

- 4 a. Solve $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2\sin[\log(1 + x)]$ (05 Marks)
- b. Solve for y : $x^2p^4 + 2xp - y = 0$. (05 Marks)
- c. Solve for x : $P = \tan\left[x - \frac{P}{1 + P^2}\right]$. (06 Marks)

Module-3

- 5 a. Obtain the partial differential equation by eliminating the arbitrary function given $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (05 Marks)
- b. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y = 0$, $z = e^y$ and $\frac{\partial z}{\partial y} = e^{-x}$. (05 Marks)
- c. Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (06 Marks)

OR

- 6 a. Obtain the partial differential equation of $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. (05 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given that $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ if $y = (2n + 1) \frac{\pi}{2}$. (05 Marks)
- c. Find the solution of heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. (06 Marks)

Module-4

- 7 a. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$. (05 Marks)
- b. Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$. (05 Marks)
- c. Obtain the relation between Beta and Gamma function $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$. (06 Marks)

OR

- 8 a. Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. (05 Marks)
- b. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (05 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$. (06 Marks)

Module-5

- 9 a. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. (05 Marks)
- b. If $f(t)$ is a periodic function of period T , then prove that $L[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$. (05 Marks)
- c. Find the inverse Laplace transform of $\frac{4s + 5}{(s + 1)^2 (s + 2)}$. (06 Marks)

OR

- 10 a. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. (05 Marks)
- b. Find $L^{-1} \left[\frac{1}{(s-1)(s^2+1)} \right]$ by using convolution theorem. (05 Marks)
- c. Solve $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 4y = e^{-t}$ with $y(0) = 0, y'(0) = 0$. (06 Marks)
