CBCS SCHEME

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21MAT21

Second Semester B.E. Degree Examination, Jan./Feb. 2023
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Evaluate $\iint_R dydx$ where R is the region bounded by the parabola $y^2 = 4x$ and line $x = \frac{1}{4}$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\ dx\ dy$ by changing the order of integration. **(07 Marks)**

c. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

OR

2 a. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{(x+y+z)}\, dz\, dy\, dx$. **(06 Marks)**

b. By changing to the polar co-ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$. **(07 Marks)**

c. Prove that $\Gamma(1/2) = \sqrt{\pi}$. **(07 Marks)**

Module-2

3 a. Find a and b such that the surfaces $ax^2 y + z = 12$ and $5x^2 - byz = 9x$ intersect orthogonally at (1, -1, 2). **(06 Marks)**

b. If $\vec{F} = (x + y + 1)\ \hat{z} + j - (x + y)\ k$, then show that $\vec{F} \cdot \text{Curl}\ \vec{F} = 0$. **(07 Marks)**

c. Show that $\vec{F} = \left(\frac{x}{x^2+y^2}\right) i + \left(\frac{y}{x^2+y^2}\right) j$. Is both solenoidal and irrotational. **(07 Marks)**

OR

4 a. If $\vec{F} = x^2 i + xy\, j$, evaluate $\int \vec{F} . d\vec{r}$ from (0, 0) to (1, 1) along
i) the line $y = x$ ii) the parabola $y = \sqrt{x}$. **(06 Marks)**

b. Evaluate using Green's theorem $\int_c e^{-x} \sin y\, dx + e^{-x} \cos y\, dy$, where C is the rectangle with vertices. (0, 0), (π, 0), (π, $\pi/2$), (0, $\pi/2$). **(07 Marks)**

c. Apply Gauss divergence theorem to evaluate $\iiint_V \text{div}\, F\, dv$ where $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the reactangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $Z = yf(x) + xg(y)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. **(07 Marks)**

c. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - \ell z)\frac{\partial z}{\partial y} = \ell y - mx$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating arbitrary constants from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial t} = e^{-2t}\cos 3x$ subject to the condition i) z(x, 0) = 0 ii) $\frac{\partial z}{\partial t}(0, t) = 0$. **(07 Marks)**

c. With usual notations derive One-dimensional heat equation. **(07 Marks)**

Module-4

7 a. Find a real root of the equation tanx + tanhx = 0 in (2, 3) by the Regula-Falsi method, correct to 2 decimal places. **(06 Marks)**

b. A function y = f(x) is given by

x :	1	1.2	1.4	1.6	1.8	2.0
y :	0.0	0.128	0.544	1.296	2.432	4.00

Find f(1.1) by using Newton's forward interpolation formula. **(07 Marks)**

c. By dividing the interval (0, π) into 6 equal parts, find the approximate value of $\int_0^\pi e^{\sin x}\,dx$ using Simpson's 1/3rd rule. **(07 Marks)**

OR

8 a. By Newton-Raphson method find the root that lies near x = 4.5 of the equation tanx – x = 0 correct to 4 decimal places. (x is in radians). **(06 Marks)**

b. Using Lagrange's interpolation method, find the value of f(x) at x = 5 given the values

x:	1	3	4	6
f(x):	3	9	30	132

(07 Marks)

c. Using Simpson's 3/8th rule, evaluate $\int_0^{0.3}\sqrt{1-8x^3}\,dx$ by taking 7 ordinates. **(07 Marks)**

Module-5

9 a. Use Taylor series method to find y(0.1) considering upto fourth degree term, given that $\frac{dy}{dx} = x - y^2$, y(0) = 1. **(06 Marks)**

b. Using Runge-Kutta method of fourth order, find y(0.1) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.1. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, compute y at x = 0.8 by applying Milne's method. **(07 Marks)**

OR

10 a. Using modified Euler's method find y(0.1) correct to four decimal places taking h = 0.1, given that $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with y(0) = 1. **(06 Marks)**

b. Use fourth order Runge-Kutta method to solve $(x+y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5 correct to four decimal places. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090, find y(0.4) correct to four decimal places by using Milne's predictor – corrector method. **(07 Marks)**

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