# CBCS SCHEME

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15MAT31

## Third Semester B.E. Degree Examination, Jan./Feb. 2023
## Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***



### Module-1

1 a. Find the Fourier series of

$$f(x) = \begin{cases} 1 + \dfrac{4x}{3} & \text{in } -\dfrac{3}{2} < x \le 0 \\ 1 - \dfrac{4x}{3} & \text{in } 0 \le x \le \dfrac{3}{2} \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$ **(08 Marks)**

b. Obtain the constant term and the first two coefficients in the Fourier cosine series for y using the following table:

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 4 | 8 | 15 | 7 | 6 | 2 |

**(08 Marks)**

**OR**

2 a. Obtain the Fourier series for

$$f(x) = \begin{cases} -K & \text{in } (-\pi, 0) \\ +K & \text{in } (0, \pi) \end{cases}.$$

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots..$ **(06 Marks)**

b. Find a half range Fourier Cosine Series for $f(x) = (x-1)^2$ in $0 \le x \le 1$. **(05 Marks)**

c. Given the following table:

| x° | 0 | 60° | 120° | 180° | 240° | 300° |
|---|---|---|---|---|---|---|
| y | 7.9 | 7.2 | 3.6 | 0.5 | 0.9 | 6.8 |

Obtain the Fourier series neglecting the terms higher than first harmonics. **(05 Marks)**

### Module-2

3 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

where 'a' is a positive real constant. Hence evaluate $\int_0^\infty \frac{\sin ax}{x} dx$. **(06 Marks)**

b. Find the Z-T of $\cosh n\theta$. **(05 Marks)**

c. Solve by using z-transforms $u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = 0 = u_1$. **(05 Marks)**

OR

4 a. Find the Fourier sine and cosine transform of $f(x) = e^{-ax}, a > 0, x \neq 0$. (06 Marks)

b. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1-|x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$ (05 Marks)

c. Given $\pi(z) = \dfrac{4z^2 - 2z}{(z-1)(z-2)^2}$. Find $u_n$. (05 Marks)

**Module-3**

5 a. Calculate the coefficient of correlation for the following data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x (Height of Father in inches) | 65 | 66 | 67 | 68 | 69 | 70 | 72 |
| y (Height of their Son in inches) | 67 | 68 | 65 | 68 | 72 | 69 | 71 |

(06 Marks)

b. Fit an exponential curve of the form $y = ae^{bx}$ by the method of least squares for the following data:

| | | | | | | |
|---|---|---|---|---|---|---|
| Number of petals | 5 | 6 | 7 | 8 | 9 | 10 |
| Number of flowers | 133 | 55 | 23 | 7 | 2 | 2 |

(05 Marks)

c. Show that a root of equation $x^3 + 5x - 11 = 0$ lies between 1 and 2. Find the root by Newton-Raphson method. (Carry out 3 iterations) (05 Marks)

OR

6 a. Obtain regression line of y on x for the following data:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 36 | 23 | 27 | 28 | 28 | 29 | 30 | 31 | 33 | 35 |
| y | 29 | 18 | 20 | 22 | 27 | 21 | 29 | 27 | 29 | 28 |

(06 Marks)

b. Fit a parabola $y = a + bx + cx^2$ for the data:

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

(05 Marks)

c. Compute real root of $x \log_{10} x - 1.2 = 0$ between 2 and 3 using Regula-Falsi method. Carry out three iterations. (05 Marks)

**Module-4**

7 a. Using suitable interpolation formula, find y(38) and y(85) for the following data:

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 40 | 50 | 60 | 70 | 80 | 90 |
| y | 184 | 204 | 226 | 250 | 276 | 304 |

(06 Marks)

b. Construct an interpolating formula for the data given below using Newton's divided difference interpolation formula.

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 2 | 4 | 5 | 6 | 8 | 10 |
| f(x) | 10 | 96 | 196 | 350 | 868 | 1746 |

(05 Marks)

c. By dividing the range into 6 equal parts, find the approximate value of $\int_0^{\pi} e^{\sin x} dx$ using Simpson's $\frac{1}{3}^{rd}$ rule. (05 Marks)

**OR**

8 a. Given Sin 45° = 0.7071, Sin 50° = 0.7660, Sin 55° = 0.8192, Sin 60° = 0.8660. Find Sin 57° using an appropriate interpolation formula. **(06 Marks)**

b. Use Lagrange's interpolation formula to find f(9), given the data:

| x | 5 | 7 | 11 | 13 | 17 |
|---|---|---|---|---|---|
| f(x) | 150 | 392 | 1452 | 2366 | 5202 |

**(05 Marks)**

c. Evaluate $\int_0^1 \frac{1}{1+x}dx$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. **(05 Marks)**

## Module-5

9 a. If $\vec{f} = (2y+3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$, evaluate the integral $\int_C \vec{f}\cdot d\vec{r}$ where 'c' is the curve $x = 2t^2$, $y = t$, $z = t^3$ from the point (0, 0, 0) to the point (2, 1, 1). **(06 Marks)**

b. Using the divergence theorem, evaluate $\int_S \vec{f}\cdot\hat{n}\,ds$ where $\vec{f} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and 's' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(05 Marks)**

c. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{(y')^2}{x^3}dx$. **(05 Marks)**

**OR**

10 a. Evaluate $\int_C xydx + xy^2dy$ by Stoke's theorem where 'c' is the square in the xy - plane with vertices (1, 0), (–1, 0), (0, 1), (0, –1). **(06 Marks)**

b. Verify the Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where 'c' is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(05 Marks)**

c. Solve the variational problem $\delta\int_1^2 [x^2(y')^2 + 2y(x+y)]dx = 0$ given $y(1) = y(2) = 0$. **(05 Marks)**

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