CBCS SCHEME

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21MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Laplace transform of

$te^{2t} - \dfrac{2\sin 3t}{t}$. **(06 Marks)**

b. Given that $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$

where $f(t + a) = f(t)$ show that $L\{f(t)\} = \dfrac{E}{S}\tan h\left(\frac{as}{4}\right)$. **(07 Marks)**

c. Using convolution theorem obtain the inverse. Laplace transform of the following function :

$\dfrac{1}{(s-1)(s^2+1)}$. **(07 Marks)**

OR

2 a. Find the inverse Laplace transform of :

$\dfrac{s+5}{s^2 - 6s + 13}$. **(06 Marks)**

b. Express the following function interms of unit step function and hence find their Laplace transform.

$f(t) = \begin{cases} 1, & 0 < t < 1 \\ t, & 1 < t \le 2 \\ t^2 & t > 2. \end{cases}$ **(07 Marks)**

c. Solve the following intial value problem by using Laplace transform :

$\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 4y = e^{-t}, y(0) = 0,\ y'(0) = 0.$ **(07 Marks)**

Module-2

3 a. Obtain Fourier series of $f(x) = \dfrac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that

$1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \text{------} = \dfrac{\pi}{4}$. **(06 Marks)**

b. Find a cosine Fourier series for $f(x) = (x - 1)^2, 0 \le x \le 1$. **(07 Marks)**

c. Obtain the Fourier series of y upto the First harmonic for the following values.

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	–1.5	0	2.8	3.4

(07 Marks)

OR

4 a. Obtain Fourier series for

$$f(x)=\begin{cases}\pi x & \text{in } 0\le x\le 1\\ \pi(2-x) & \text{in } 1\le x\le 2\end{cases}.$$ (06 Marks)

b. Obtain the sine half range series for the function :

$$f(x)=1-\left(\frac{x}{\pi}\right)\text{in } \ 0\le x\le \pi.$$ (07 Marks)

c. The following values of y and x are given. Find Fourier series of upto first harmonics.

x	0	2	4	6	8	10	12
y	9.0	18.2	24.4	27.8	27.5	22.0	9.0

(07 Marks)

Module-3

5 a. If $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|\ge 1\end{cases}$. Find Fourier transform of f(x) and hence find the value of

$$\int_0^\infty \frac{x\cos x-\sin x}{x^3}\,dx.$$ (06 Marks)

b. Find the Fourier sine transform of $f(x)=e^{-|x|}$ and hence evaluate

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx,\ m>0\cdot$$ (07 Marks)

c. Solve by using Z-Transforms $U_{n+2}+2U_{n+1}+U_n=n$ with $U_0=0=U_1$. (07 Marks)

OR

6 a. Obtain the Fourier cosine transform of the function :

$$f(x)=\begin{cases}4x, & 0<x<1\\ 4-x, & 1<x\le 4\\ 0, & x>4.\end{cases}$$ (06 Marks)

b. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$ (07 Marks)

c. Compute the inverse Z-transform of $\dfrac{3z^2+2z}{(5z-1)(5z+2)}$. (07 Marks)

Module-4

7 a. Classify the following partial differential equations :

i) $x^2u_{xx}+(1-y^2)u_{yy}=0,\ -\infty<x<\infty,-1<y<1$

ii) $(1+x^2)u_{xx}+(5+2x^2)u_{xt}+(4+x^2)u_{tt}=0$

iii) $(x+1)u_{xx}-2(x+2)u_{xy}+(x+3)u_{yy}=0$. (10 Marks)

b. Solve $u_t = u_{xx}$ subject to the conditions u(0, t) = 0 = u(1, t) and u(x, 0) = sin(πx) by taking h = 0.2 for 5 levels. Further write down the following values from the table

i) u(0.2, 0.04)

ii) u(0.4, 0.08)

iii) u(0.6m 0.06). (10 Marks)

OR

8 a. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square Mesh with boundary values as shown. Find the iterative values of u_i(1 to 9) to the nearest integer.

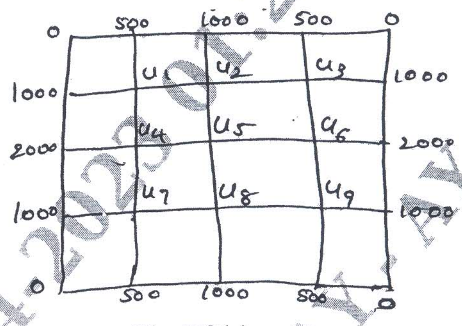

Fig.Q8(a) **(10 Marks)**

b. Solve $25u_{xx} = u_{tt}$ at the pivotal points given $u(0, t) = 0 = u(5, t)$, $u_t(x, 0) = 0$ and

$$u(x,0) = \begin{cases} 20x, & 0 \le x \le 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases}$$ by taking $h = 1$ compute $u(x, t)$ for $0 \le t \le 1$. **(10 Marks)**

Module-5

9 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$ compute $y(0.2)$ using fourth order Runge – Kutta method. **(06 Marks)**

b. Derive the Euler's equation. **(07 Marks)**

c. Find the extremal of the functional.

$$\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)dx .$$ **(07 Marks)**

OR

10 a. Obtain the solution of the equation $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of y(1.4) by applying Milne's method using following data :

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

(06 Marks)

b. Find the curve on which the functional $\int_0^1 \left[[y']^2 + 12xy\right]dx$ with $y(0) = 0$ and $y(1) = 1$ can be determined. **(07 Marks)**

c. Prove that the shortest distance between two points in a plane is straight line. **(07 Marks)**

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