CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Laplace transform of:
i) $(3t+4)^2 + 5^t$
ii) $e^{-t}\cos^2 3t$
iii) $\dfrac{\cos at - \cos bt}{t}$ **(10 Marks)**

b. Given $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$ where $f(t+a) = f(t)$, show that $L[f(t)] = \dfrac{E}{S}\tanh\ (as/4)$. **(05 Marks)**

c. Employ Laplace transform to solve the equation: $y'' + 5y' + 6y = 5e^{2t}$, taking $y(0) = 2$, $y'(0) = 1$. **(05 Marks)**

OR

2 a. Find the Inverse Laplace transform of:
i) $\dfrac{(s+2)^2}{s^6}$ ii) $\dfrac{s+1}{s^2+6s+9}$ iii) $\dfrac{3s+2}{s^2-s-2}$ **(10 Marks)**

b. Express $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$ in terms Heaviside's unit step function and hence find its Laplace transform. **(05 Marks)**

c. Find the Laplace transform of $\dfrac{s}{(s^2+a^2)^2}$ using convolution theorem. **(05 Marks)**

Module-2

3 a. Find the Fourier series expansion of $f(x) = x - x^2$ in $-\pi \le x \le \pi$. Hence deduce that $\dfrac{x^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + ...$ **(07 Marks)**

b. Find the half-range cosine series of $f(x) = 2x-1$ in the interval $0 < x < 1$. **(06 Marks)**

c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

x^o	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(07 Marks)

OR

4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-l, l)$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$. **(07 Marks)**

b. Find the sine half range series of $f(x) = \begin{cases} \frac{1}{4} - x & \text{in} \quad 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in} \quad \frac{1}{2} < x < 1 \end{cases}$ **(06 Marks)**

c. The following table gives the variations of a periodic current A over a certain period T:

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a constant part of 0.75amp. in the current A, and obtain the amplitude of the first harmonic. **(07 Marks)**

Module-3

5 a. If $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ find the Fourier transform of f(x) and hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3} dx$. **(07 Marks)**

b. Find the Fourier sine and cosine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$. **(06 Marks)**

c. Solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$ by using z-transform. **(07 Marks)**

OR

6 a. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx$, $m > 0$. **(07 Marks)**

b. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{3z^2 + 2z}{(5z - 1)(5z + 2)}$. **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ using Taylor's series method considering upto fourth degree terms and find the value of y(0.1). **(07 Marks)**

b. Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$ taking $h = 0.2$. **(06 Marks)**

c. Apply Milne's method to compute y(1.4) correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the data: $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. **(07 Marks)**

OR

8 a. Using modified Euler's method find y(20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 taking h = 0.2. **(07 Marks)**

b. Use Fourth order Runge-Kutta method to compute y(1.1) given that $\frac{dy}{dx} = xy^{1/3}$, y(1) = 1. **(06 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) using Adams – Bashforth predictor-corrector method. **(07 Marks)**

Module-5

9 a. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, y(0) = 1, y′(0) = 0, evaluate y(0.1) using Runge-Kutta method of 4th order. **(07 Marks)**

b. Find the external of the functional $\int_{x_1}^{x_2}\left(y^{1^2} - y^2 + 2y\sec x\right)dx$. **(06 Marks)**

c. Derive Euler's equation in the standard form:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0.$$

(07 Marks)

OR

10 a. Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values:

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

(07 Marks)

b. Find the external of the functional $\int_0^{\pi/2}(y^2 - y^{1^2} - 2y\sin x)\,dx$ under the end conditions y(0) = 0, y(π/2) = 0. **(06 Marks)**

c. Prove that the geodesics on a plane are straight lines. **(07 Marks)**

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