# BRIDGE COURSE

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MATDIP301

## Third Semester B.E. Degree Examination, April 2023
## Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the modulus and amplitude of the complex number $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Express the complex number $Z = \frac{(3+i)(1-3i)}{2+i}$ in the form of x + iy. **(06 Marks)**

c. Explain $\sqrt{3}+i$ in polar form and hence find the modulus and amplitude. **(08 Marks)**

2 a. Find the $n^{th}$ derivative of $e^{ax}$. **(06 Marks)**

b. Find the $n^{th}$ derivative of $\cos^2 3x$. **(06 Marks)**

c. If $y = e^{m\sin^{-1}x}$, then prove that $(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$. **(08 Marks)**

3 a. Find the angle between the radius vector and the tangent to the curve $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Obtain the Maclaurin's series expansion of the function sinx up to the term containing $x^4$. **(06 Marks)**

c. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(08 Marks)**

4 a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. **(06 Marks)**

b. If $u = \log(x^3 + y^3)$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. **(06 Marks)**

c. If $u = x + y$, $v = y + z$, $w = z + x$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(08 Marks)**

5 a. Evaluate $\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy\,dy\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi/6} \sin^6 3x\,dx$. **(06 Marks)**

c. Obtain the reduction formula for $\int \sin^n x\,dx$, where n is positive integer. **(08 Marks)**

6 a. Evaluate $\int_{x=0}^{2}\int_{y=1}^{3}\int_{z=1}^{2} xy^2z.dzdydx$. (06 Marks)

b. Evaluate $\iint_R xydxdy$, where R is the region in positive Quadrant for which $x + y \le 1$. (06 Marks)

c. With usual notations, prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (08 Marks)

7 a. Solve: $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. (06 Marks)

b. Solve: $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. (06 Marks)

c. Solve: $(2x + y + 1)dx + (x + 2y + 1)dy = 0$. (08 Marks)

8 a. Solve: $(D^3 - 6D^2 + 11D - 6)\,y = 0$. (06 Marks)

b. Solve: $(D^2 + 6D + 9)\,y = 0$. (06 Marks)

c. Solve: $(D^3 - 2D^2 + 4D - 8)\,y = 0$. (08 Marks)

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