CBCS SCHEME

USN

17MAT11

First Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\sin x \ \sin 2x \ \sin 3x$. **(06 Marks)**

b. If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(07 Marks)**

c. With usual notations, prove that $\tan\varphi = r.\dfrac{d\theta}{dr}$. **(07 Marks)**

OR

2 a. Find the angle between radius vector and tangent to the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

b. Find the pedal equation to the curve $r = a(1 - \cos\theta)$. **(07 Marks)**

c. Find the radius curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on it. **(07 Marks)**

Module-2

3 a. Expand $\tan x$ in powers of $\left(x - \dfrac{\pi}{4}\right)$ upto third degree term. **(06 Marks)**

b. Evaluate $\lim\limits_{n\to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$. **(07 Marks)**

c. If $u = \dfrac{x}{y+z} + \dfrac{y}{z+x} + \dfrac{z}{x+y}$, prove that $x\cdot\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$. **(06 Marks)**

b. If $u = \sin^{-1}\left[\dfrac{x^3 + y^3}{x+y}\right]$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\tan u$. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $J\left(\dfrac{u.v.w}{x,y,z}\right)$. **(07 Marks)**

Module-3

5 a. A particle moves along a curve with parametric equations $x = t - \dfrac{t^3}{3}$, $y = t^2$ and $z = t + \dfrac{t^3}{3}$, where t is the time. Find velocity and acceleration at any time 't' and also find their magnitudes at t = 3. **(06 Marks)**

b. Find the unit normal vector to the surface $x^2yz + xy^2z + xyz^2 = 3$ at (1, 1, 1). **(07 Marks)**

c. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

OR

VTU-27-07-2023 08:46:22am AY - AY - AY - AY 27/07/2023 09:03:50am AY - AY - AY

6 a. If $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$, find $\frac{d}{dt}(\vec{A}\times\vec{B})$. **(06 Marks)**

b. Show that the vector $\vec{F} = (3x^2 - 2yz)\hat{i} + (3y^2 - 2zx)\hat{j} + (3z^2 - 2xy)\hat{k}$ is irrotational. Also find the scalar φ such that $\vec{F} = \text{grad}\,\varphi$. **(07 Marks)**

c. Prove that $\text{div}(\varphi\vec{A}) = (\text{grad}\,\varphi).\vec{A} + \varphi(\text{div}\,\vec{A})$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x dx$. **(06 Marks)**

b. Solve $(2x^3 - xy^2 - 2y + 3)dx = (x^2y + 2x)dy = 0$. **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is a parameter. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 min, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Module-5

9 a. Find the rank of matrix

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$$

(06 Marks)

b. Solve $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ by Gauss Elimination method. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix

$$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

Take $[1\ \ 0\ \ 0]^T$ as initial Eigen vector. Use Rayleigh's power method. Carry out 4 iterations. **(07 Marks)**

OR

10 a. Solve $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$ by Gauss-Seidel method. Carryout 3 iterations. **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to Canonical form. **(07 Marks)**

* * * * *