CBCS SCHEME

USN

21MAT11

First Semester B.E. Degree Examination, June/July 2023
Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$. (06 Marks)

b. Find the angle between the curves $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$. (07 Marks)

c. Show that for the curve $r(1 - \cos\theta) = 2a$, ρ^2 varies as r^3. (07 Marks)

OR

2 a. Find the pedal equation of the curve $\dfrac{2a}{r} = 1 + \cos\theta$. (06 Marks)

b. Find the angle between the curves $r = a\log\theta$ and $r = \dfrac{a}{\log\theta}$. (07 Marks)

c. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$. (07 Marks)

Module-2

3 a. Expand the function $\sqrt{1+\sin 2x}$ by Maclaurin's series up to the term containing x^4. (06 Marks)

b. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then show that $6u_x + 4u_y + 3u_z = 0$. (07 Marks)

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. Show that $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$. (07 Marks)

OR

4 a. Evaluate : $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x}{3}\right)^{1/x}$. (06 Marks)

b. If $u = \tan^{-1}(y/x)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$, find the total derivative $\dfrac{du}{dt}$ using partial differentiation. (07 Marks)

c. Find the extreme values of $x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

Module-3

5 a. Solve : $\dfrac{dy}{dx} - y\tan x = y^2\sec x$. (06 Marks)

b. Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$ where λ is a parameter. (07 Marks)

c. Solve : $p^2 + 2py\cot x - y^2 = 0$. (07 Marks)

OR

6 a. Solve : $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. **(06 Marks)**

b. Water at temperature 10°C takes 5 minutes to warm up to 20°C at a room temperature of 40°C. Find the temperature of the water after 20 minutes. **(07 Marks)**

c. Find the general solution of the equation $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form by taking the substitution $X = x^2$, $Y = y^2$. **(07 Marks)**

Module-4

7 a. Solve : $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$. **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$. **(07 Marks)**

c. Solve by using method of variation of parameters $y'' - 2y' + y = \frac{e^x}{x}$. **(07 Marks)**

OR

8 a. Solve : $y'' + 2y' + y = e^{3x}$. **(06 Marks)**

b. Solve : $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$. **(07 Marks)**

c. Solve : $(D^2 + 4)y = x^2$. **(07 Marks)**

Module-5

9 a. Find the rank of a matrix by reducing in to echelon form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

(06 Marks)

b. Solve the system of equations by Gauss-Jordan method: $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$. **(07 Marks)**

c. Solve the system of equations by Gauss-Seidel iterative method : $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. Perform 3 iterations by choosing (0, 0, 0) as initial approximation. **(07 Marks)**

OR

10 a. For what values of λ and μ, the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has (i) no solution (ii) Unique solution (iii) Infinitely many solutions. **(06 Marks)**

b. Solve the system of equations by Gauss elimination method: $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$. **(07 Marks)**

c. Using Rayleigh's power method, find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking $[1 \quad 0 \quad 0]^T$ as initial eigen vector. Carry out 5 iterations. **(07 Marks)**

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