CBCS SCHEME

USN | | | | | | | | | |

18MAT31

Third Semester B.E. Degree Examination, June/July 2023

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find $L\left(\dfrac{\cos at - \cos bt}{t}\right)$. **(06 Marks)**

b. Express the function in terms of unit step function and hence find Laplace transform of

$$f(t) = \begin{cases} \sin t & 0 < t < \frac{\pi}{2} \\ \cos t & \frac{\pi}{2} < t < \pi \end{cases}$$ **(07 Marks)**

c. Solve $y''(t) + 4y'(t) + 3y(t) = e^t$, $y(0) = y'(0) = 1$ by using Laplace transform method. **(07 Marks)**

OR

2 a. Find : (i) $L^{-1}\left(\log\left(\dfrac{s+b}{s+a}\right)\right)$ (ii) $L^{-1}\left(\dfrac{s+3}{s^2 - 4s + 13}\right)$ **(06 Marks)**

b. Find $L^{-1}\left(\dfrac{s}{(s^2 + a^2)^2}\right)$ by using convolution theorem. **(07 Marks)**

c. Given $f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$

where $f(t) = f(t + 2a)$ then show that $L(f(t)) = \dfrac{1}{s^2}\tan h\left(\dfrac{as}{2}\right)$ **(07 Marks)**

Module-2

3 a. Obtain Fourier series for $f(x) = \dfrac{\pi - x}{2}$, $0 < x < 2\pi$. **(06 Marks)**

b. Find Fourier series for $f(x) = 2x - x^2$, $0 < x < 2$. **(07 Marks)**

c. Find half range Fourier cosine series for

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$ **(07 Marks)**

OR

4 a. Find Fourier series for $f(x) = |x|$, $-\pi < x < \pi$. **(06 Marks)**

b. Obtain Fourier series for $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$. **(07 Marks)**

c. Find the Fourier series upto first harmonic from the following table:

x	0	1	2	3	4	5
y = f(x)	4	8	15	7	6	2

(07 Marks)

Module-3

5 a. Find Fourier transform of f(x), given:

$$f(x)=\begin{cases}1, & |x|\le 1\\ 0, & |x|>1\end{cases} \quad \text{and hence deduce that } \int_0^\infty \frac{\sin x}{x}\,dx=\frac{\pi}{2}.$$ **(06 Marks)**

b. Find the Fourier cosine transform of

$$f(x)=\begin{cases}4x & 0<x<1\\ 4-x & 1<x<4\\ 0 & x>4\end{cases}$$ **(07 Marks)**

c. Solve $u_{n+2}+4u_{n+1}+3u_n=3^n$, given $u_0=0$, $u_1=1$ using Z - transform. **(07 Marks)**

OR

6 a. Find the Fourier sine transform of $e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx$. **(06 Marks)**

b. Find Z-transform of $\cos n\theta$ and $a^n\cos n\theta$. **(07 Marks)**

c. Obtain the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. **(07 Marks)**

Module-4

7 a. Find the value of y at x = 0.1 and x = 0.2 given $\frac{dy}{dx}=x^2y-1$, y(0) = 1 by using Taylor's series method. **(06 Marks)**

b. Compute y(0.1), given $\frac{dy}{dx}=\frac{y-x}{y+x}$, y(0) = 1 taking h = 0.1, by using Runge-Kutta 4th order method. **(07 Marks)**

c. Find the value of y at x = 0.4, given $\frac{dy}{dx}=2e^x-y$ with initial conditions y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04, y(0.3) = 2.09 by using Milne's predictor and corrector method. **(07 Marks)**

OR

8 a. Using modified Euler's method, find the value of y at x = 0.1, given $\frac{dy}{dx}=-xy^2$, y(0) = 2 taking h = 0.1. **(06 Marks)**

b. Solve $\frac{dy}{dx}=3e^x+2y$, y(0) = 0 at x = 0.1 taking h = 0.1, by using Runge-Kutta 4th order method. **(07 Marks)**

c. Find the value y at x = 0.8 given $\frac{dy}{dx}=x-y^2$ and

x	0	0.2	0.4	0.6
y	0	0.0200	0.0795	0.1762

By using Adam's Bashforth predictor and corrector method. **(07 Marks)**

Module-5

9 a. Solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 given x = 0, y = 1 and $\frac{dy}{dx} = 0$ by using Runge-Kutta method. **(07 Marks)**

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

c. Find the extremal of the function $\int_0^1 [(y')^2 + 12xy]dx$ with y(0) = 0 and y(1) = 1. **(07 Marks)**

OR

10 a. Find the value of y at x = 0.8, given $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$ and

x	0	0.2	0.4	0.6
y	1	0.2027	0.4228	0.6841
y′	1	1.041	1.179	1.468

by using Milne's method. **(07 Marks)**

b. Prove that the shortest between two points in a plane is a straight line. **(06 Marks)**

c. Find the curve on which the functional $\int_0^1 [x + y + (y')^2]dx$ with y(0) = 1, y(1) = 2. **(07 Marks)**

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